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Journal of the

SURVEYING AND MAPPING DIVISION

Proceedings of the American Society of Civil Engineers

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Journal of the
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SELF-CHECKING METHOD OF COMPUTING CURVE ELEVATIONS

By Charles M. Lamont,[1] M. ASCE

SYNOPSIS

This paper describes a rapid self-checking method of computing vertical parabolic curve elevations using the conventional type mechanical calculating machine.

INTRODUCTION

Several methods of computing vertical-curve elevations are being used. However, it is believed that the proposed method, since it is self-checking and so well adapted to use with the ordinary calculating machine, is preferred rather than the usual laborious methods requiring squaring, multiplication, and division of large figures. This more simplified method is especially useful in bridge work, with skewed structures on complicated layouts which involve many time consuming and lengthy calculations to arrive at elevations for odd stations.

PRINCIPLES

For the parabola used in highway work:

1. The slope changes linearly as the distance from the V. P. C., and the rate of change is given by

$$R = \frac{G_2 - G_1}{L} \quad \dots\dots\dots\dots\dots\dots\dots \quad (1)$$

Note.—Discussion open until July 1, 1960. Separate Discussions should be submitted for the individual papers in this symposium. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Surveying and Mapping Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. SU 1, February, 1960.

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2a. At any point between the V. P. C. and the high or low point, the slope of the local tangent is given by

$$S = G_1 \pm \frac{(G_2 - G_1)}{L} \times \text{Distance in stations from the V.P.C. to the point} \quad \text{(2a)}$$

2b. At any point between the high or low point and the V.P.T., the local tangent slope is given by:

$$\frac{(G_2 - G_1)}{L} \times \text{Distance in stations from the high or low point to the point} \quad \text{(2b)}$$

3. The gradient, g, between any two adjacent points is the mean of the two local tangent slopes, S (Fig. 1). These gradients are:

$$g_1 = \frac{G_1 + S_1}{2} \quad \text{(3a)}$$

$$g_2 = \frac{S_1 + S_2}{2} \quad \text{(3b)}$$

$$g_3 = \frac{S_2 + S_3}{2} \quad \text{(3c)}$$

and

$$g_4 = \frac{S_3 + G_2}{2} \quad \text{(3d)}$$

Then the elevation at each point is

Elev. of Pt. (1) = Elev. of V.P.C. + (d_1 x g_1)

Elev. of Pt. (2) = Elev. of Pt. (1) + (d_2 x g_2)

Elev. of Pt. (3) = Elev. of Pt. (2) + (d_3 x g_3)

Elev. of V.P.T. = Elev. of Pt. (3) + (d_4 x g_4)

This last elevation, the elevation of the V.P.T., should check the given elevation of the V.P.T. This is a check on all of the elevations computed.

ILLUSTRATIVE PROBLEM

To illustrate this method, the computations for the vertical curve in Fig. 2 will be performed to illustrate how they can be computed on any standard calculating machine and recorded as shown in Table 1.

Step I.—Compute the local tangent slope, S_1, S_2, S_3, etc. for each point.

1. Compute the rate of change of slope per 100 ft station:

$$R = \frac{-2.45 - 0.75}{8} = 0.40000\%$$

2. Set the decimal points on the machine: Three decimal places are set into the upper dials of the carriage (multiplier dials). Ten decimal places are set

into the lower dials of the carriage (product dials). Five decimal places are set into the keyboard. These decimal points will be held throughout all of the calculations.

3. The figures are now set into the machine in the following order:

a. G_1, 0.7500000000, into the product (lower) dials.
b. The station of the V.P.C., 8675.000, into the multiplier (upper) dial.
c. R, 0.40000, into the keyboard.

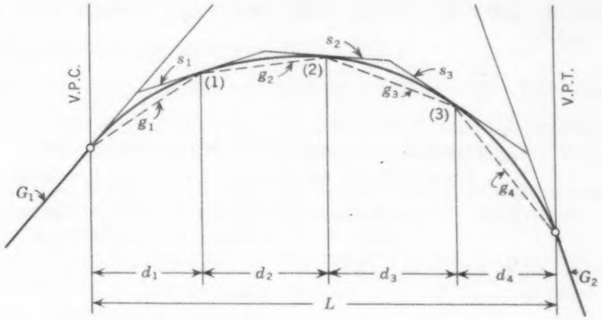

FIG. 1

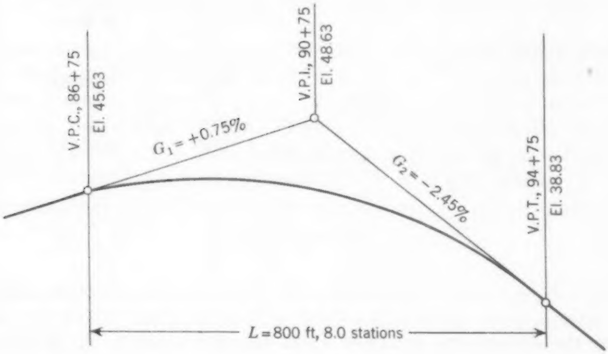

FIG. 2

4. Now by visualizing the shape of curve, the positive-negative multiplier lever is set on negative multiplication since the slopes will be decreasing numerically as the stations increase. (This is accomplished on some calculating machines, not having the positive-negative multiplication lever feature, by the use of red figures in the multiplier dials).

5. The multiplier dials are changed to the station of each point, the keyboard remaining unchanged, and the local tangent slopes, S, are read from the product dials and recorded in Column 3.

The high point of the curve is reached when the local tangent slope in the product dial reads zero or as close to zero as the product dials will read. Then, visualizing the shape of the curve, the positive-negative multiplication lever is set to positive multiplication, since from the high point on an increase in station produces a numerical increase in the slope. When the V.P.T. station, 94+75, is reached the product dials should indicate the local tangent slope, G_2, or 2.45% for this problem (or very close to it). This is a check on the operation of the calculating machine thus far.

Step II.—Compute the point-to-point gradients, g_1, g_2, g_3, etc. This is done in the following order by mental subtraction and addition and recorded in Columns 4 and 5:

TABLE 1.—ILLUSTRATIVE COMPUTATIONS

Station	Point	Local tangent slope, S	(1/2 Difference) Difference	Point-to-point gradient, g	Point elevation
86+75	V.P.C.	G_1 = +0.75%			45.63
			(0.05000) 0.10000	+0.70000	
87+00		+0.65000			45.805
			(0.02056) 0.04113	+0.62943	
87+10.283		+0.60887			45.870
			(0.30443) 0.60887	+0.30443	
88+62.500	H.P.	0			46.333
			(0.08984) 0.17968	-0.08984	
89+07.420		-0.17968			46.293
			(1.13516) 2.27032	-1.31484	
94+75	V.P.T.	G_2 = -2.45%			38.83 (38.830)

1. Obtain the difference between local tangent slopes, S, in Column 3 and record this difference in Column 4:

$$0.75 - 0.65000 = 0.10000$$

$$0.65000 - 0.60887 = 0.04113$$

$$0.60887 - 0 = 0.60887, \text{ etc.}$$

2. Obtain 1/2 of this difference and enter in parentheses above the figure in Column 4:

$$1/2 \text{ of } 0.10000 = 0.05000$$

$$1/2 \text{ of } 0.04113 = 0.02056, \text{ etc.}$$

3. Now the mean of adjacent slopes, g, is obtained by adding the figure in parentheses computed in 2 to the smaller of the adjacent local tangent slopes:

$$0.65 \quad + 0.0500 \ = +0.70000$$

$$0.60887 + 0.02056 = +0.62943, \text{ etc.}$$

This is recorded in Column 5.

The gradient is given the proper algebraic sign by visualizing the shape of curve and is positive from V.P.C. to the high point and negative beyond the high point.

Step III.—Compute the elevations of each point.

1. The figures are set into the calculating machine in the following order:

   a. The elevation of the V.P.C., 45.6300000000, into the product dials.
   b. The station of the V.P.C., 8675.000, into the multiplier dials.
   c. The gradient, g_1, to +0.70000, into the keyboard.

2. The positive-negative multiplication lever is set to positive multiplication since g_1, +0.70000, is positive.

3. With g_1 in the keyboard, the multiplier dial is changed to the next point station, 8700.000, and the elevation at this station, 45.805, is read from the product dial and recorded in Column 6.

4. Now the keyboard is cleared and g_2, + 0.62943, is set into the keyboard, the other dials remaining unchanged.

5. The multiplier dial is now changed to the next point station, 8710.283, and the elevation at this station, 45.870, is read from the product dial and recorded in Column 6.

6. The above operation is repeated for each of the other point stations until the V.P.T. station, 9475.000, is reached. The product dial should read the elevation of the V.P.T. originally given, 38.830 (or very close to it). This is a check on all of the computations. The algebraic sign of the gradient, g, changes to negative after the high point is reached and the positive-negative multiplication lever must be changed to negative multiplication.

CONCLUSIONS

The example using a convex type curve was used, however, a concave type curve may be computed just as easily, always visualizing the shape of the curve when computing.

This method of computing provides some very useful information. The local tangent slopes, S, are useful in obtaining slopes for bevel plates, expansion devices, bearings and other details, and also indicates direction of surface drainage. The stations of the high and low points are also useful for surface drainage information.

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THE CALCULATING MACHINE IN COORDINATE GEOMETRY

By Bela Vadasz,[1] M. ASCE

SYNOPSIS

The paper systematizes the solution of common engineering problems in coordinate geometry. Surveying and layout computations, such as intersection, reverse curves, and the three-point problem are analysed and tabulated for solution on conventional calculators. Typical problems and the method of checking results are liberally exemplified.

INTRODUCTION

Most engineers engaged in the solution of problems in coordinate geometry use the office calculator to achieve the necessary accuracy. Many have developed methods for solution of the most recurrent problems. The methods of this article go somewhat further. First, from fundamental relations of coordinate geometry, there are developed abbreviated techniques for solution of six basic problems, termed "fundamental operations." Then the application of these few highly systematized techniques to a wide variety of typical problems is illustrated. Before developing these techniques, however, it is necessary to make a few introductory remarks applicable to the balance of the discussion.

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Desk calculators are generally classified as semi-automatic or automatic. On the semi-automatic machine, division is done automatically but multiplication must be performed digit by digit by the operator. On the automatic calculator both multiplication and division are performed automatically. The descriptions which follow relate to the semi-automatic machine but are readily adaptable to the automatic type.

Lack of uniformity in the terminology used by the various manufacturers necessitates a brief definition of terms. The dial in which the results of addition or multiplication appear, will be referred to as the product dial. The dial

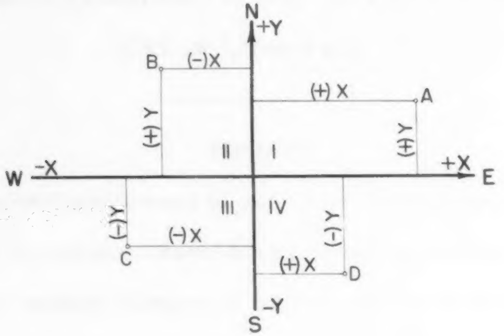


FIG. 1.

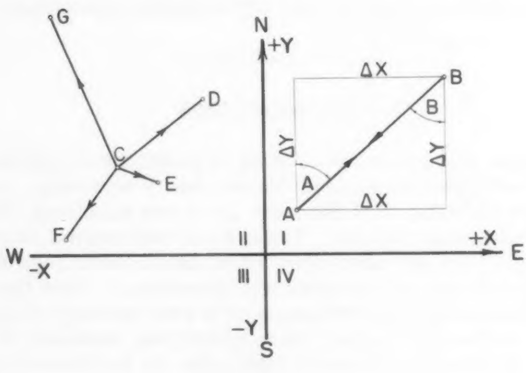


FIG. 2.

which records the number of cycles of operation will be called the counter dial. Most machines have a device for reversing the operation of this dial, so that, for example, the use of the plus-key which will normally increase the quantity shown in the counter, will, instead, decrease that quantity. When it is necessary to alter the operation of the counter, a direction will be given for setting the counter control.

All machines have a device which saves the keyboard entry so that it may be added repeatedly so long as the plus-key is depressed. Unless otherwise stated it will be assumed that the machine is set for repeated cycling.

Figs. 3 and 4 illustrate two tabular forms that will be found particularly useful in organizing the calculations.

Symbols.—Capital letters indicate points, bearings, and coordinates. Lower case letters indicate distances. The letter Δ shows differences.

FUNDAMENTAL PRINCIPLES

In the interest of establishing a common basis of reference, it may be well to restate the principles of coordinate geometry to be used.

Coordinate System.—Any kind of rectangular coordinate system may be used. In the "N-E" coordinate system, to which the following descriptions refer, the positive "X" axis faces east and the positive "Y" axis, north (Fig. 1). The positive "X" axis may, of course, face any practically chosen direction so long as the positive "Y" axis forms a right angle counterclockwise to it.

Sign and Bearing.—Both coordinates of each point have signs, dependent upon the quadrant (I, II, III, IV) in which the point occurs (Fig. 1).

As is well known, the "bearing" always means the acute angle measured from the North-South axis (Fig. 2).

TABLE 1.—SIGNS OF COORDINATES

Signs of Coordinates		Quadrant
Y	X	
+	+	I
+	-	II
-	-	III
-	+	IV

Signs of coordinates are interrelated with quadrant and bearing. The quadrant of the point refers to the location of a point and must be carefully distinguished from the direction of the bearing. The signs of the coordinates of a point describe the quadrant in which the point appears. (Fig. 1). The direction of the bearing, on the other hand, refers to a straight line. The signs of the coordinate-differences of two points on a line, describe the bearing as N-E, N-W, S-W or S-E (Fig. 2).

Table 1 illustrates the signs of coordinates in the several quadrants.

Direction of the bearing is illustrated in Fig. 2, wherein:

A, B points are in quadrant I.

AB bearing: N-E ($\Delta Y = Y_B - Y_A : +$; $\Delta X = X_B - X_A : +$)

BA bearing: S-W ($\Delta Y = Y_A - Y_B : -$; $\Delta X = X_A - X_B : -$)

C, D, E, F, G points are in quadrant II.

CD bearing: N-E ($\Delta Y : +$; $\Delta X : +$)
CE bearing: S-E ($\Delta Y : -$; $\Delta X : +$)
CF bearing: S-W ($\Delta Y : -$; $\Delta X : -$)
CG bearing: N-W ($\Delta Y : +$; $\Delta X : -$)

The magnitude of the bearing angle can be determined from the following formula:

$$\frac{X_B - X_A}{Y_B - Y_A} = \frac{\Delta X}{\Delta Y} = \tan A$$

For each of the possible directions of bearing, the corresponding signs of coordinate differences are shown in Table 2, together with the appropriate signs of the trigonometric functions.

FUNDAMENTAL OPERATIONS

Fundamental Operation 1: Computation of Bearings and Distances.— Form # 1 (shown in Fig. 3) is recommended for this computation.

Given: Points A and B by their coordinates Y_A, X_A, Y_B, X_B.

Sought: a. Bearing A; b. Distance $\overline{AB}$ = d.

a. Computation of the bearing:

$$\text{The formula: } \tan A = \frac{X_B - X_A}{Y_B - Y_A} = \frac{\Delta X}{\Delta Y}$$

The order of the operation: always subtract the coordinates of the commencing point (A in this case) from those of the terminal point (B) of the straight line.

TABLE 2.—SIGNS FOR COORDINATE DIFFERENCES AND TRIGONOMETRIC FUNCTIONS

Direction of Bearing	Signs of Coordinate Differences		Signs of Trigonometric Functions			
	ΔY	ΔX	sin	cos	tan	cot
N - E	+	+	+	+	+	+
N - W	+	-	-	+	-	-
S - W	-	-	-	-	+	+
S - E	-	+	+	-	-	-

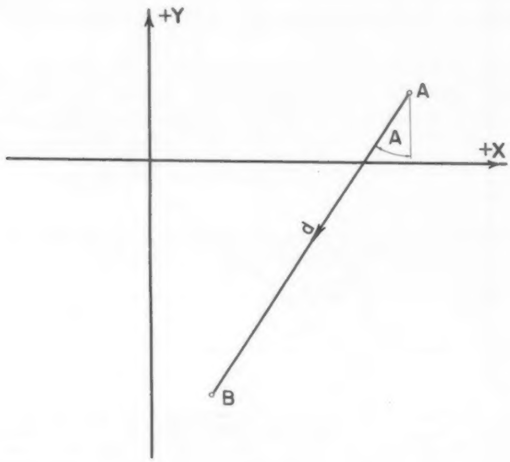
The procedure is as follows:

1. After entering the coordinates of the commencing point in the form (see example, Fig. 3), enter the coordinates of the terminal point. If this order is followed consistently, the computation can be made more automatic. Moreover, if the computation sheets are reviewed later, it will be clear that the bearing obtained is the AB, not the BA bearing.

2. Since it is necessary to subtract the coordinates of the commencing point algebraically from those of the terminal point, the signs of the commencing point coordinates must be changed. It is recommended that the changed signs be shown with red pencil.

3. If the sign of Y_A is identical, after the change, with the sign of Y_B, add the two coordinates giving to the sum the common sign. If the signs, after the change, are different, subtract the smaller figure from the larger one, giving to the difference, the sign of the larger figure. Do the same with the X coordinates.

4. Divide ΔX by ΔY. The quotient is tan A. The signs of ΔY and ΔX determine the direction of the bearing (see Table 2).



COMPUTATION OF BEARINGS AND DISTANCES						Form #1
		Y		X		
A	⊖ +	86.16	⊖ +	314.40	tan A = 0.641 9829	S 32° 41' 59" W
B	-	287.23	+	74.69		
B–A	-	373.39	-	239.71		d = 443.71

⊕, ⊖ INDICATE RED SIGNS

FIG. 3.

b. Computation of the distance.

$$d = \sqrt{\Delta Y^2 + \Delta X^2}$$

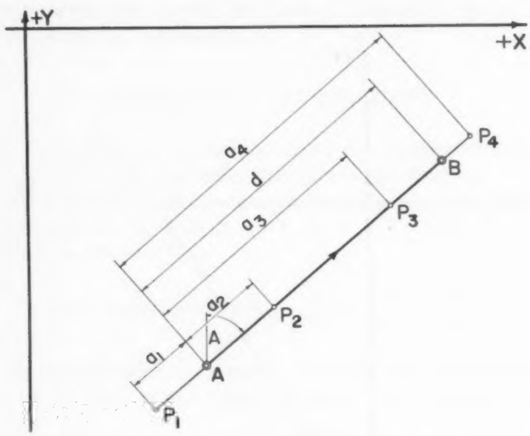
The procedure is as follows:

1. Multiply ΔY by itself.

2. Clearing all but the product dial, square ΔX. The sum of the squares will be recorded automatically in the product dial.

3. Extract square root of answer showing in the product dial.

A common machine method of extracting square root is illustrated in the Appendix.



COORDINATES OF LINE-POINTS									Form #2
1	2	3	4	5	6	7	8	9	10
PT.	DISTANCE	OFFSETS		$\frac{\Delta Y}{d}$ = cos A		$\frac{\Delta X}{d}$ = sin A		COORDINATES	
		RIGHT	LEFT	DIFF. OF DIST.		DIFF. OF OFFS.		Y	X
		+	−	+	−	+	−		
				0.648 0326		0.761 6088			
				⊖	⊕				
A	0							⊕ −411.41	⊖ +213.18
P_1	−82.06				82.06			−464.59	+150.68
P_2	108.27			190.33				−341.25	+295.64
P_3	297.45			189.18				−218.65	+439.72
P_4	427.12			129.67				−134.62	+538.48
B	378.16				48.96			−166.35	+501.19
				509.18	131.02			+ 245.06	+288.01
				− 131.02					
				378.16					
				⊖	⊕				

⊖, ⊕ INDICATE RED SIGNS

FIG. 4.

Fundamental Operation 2: Coordinates of Line-Points.—Fig. 4 is used to illustrate the following:

Given: Points A and B with their coordinates Y_A, X_A, Y_B and X_B; The distances a_1, a_2, a_3 and a_4, measured along AB from A.

Sought: Coordinates of line-points P_1, P_2, P_3 and P_4, on the line determined by points A and B.

The procedure systematizes solution of the following formulae:

$$Y_P = Y_A + a \cos A$$

$$X_P = X_B - (d-a) \sin A$$

Form # 2 (Fig. 4) will be used.

The procedure is as follows:

1. Enter "a" distances in Col. 2, as shown in Fig. 4. In the illustration a_1 is a negative number because point P_1 appears before A rather than between A and B.
2. Enter Y_A, X_A, Y_B, X_B coordinates in Cols. 9 and 10.
3. Compute ΔY, ΔX and d, unless they have already been determined in balancing a traverse. Fundamental Operation 1 may be used. Enter distance d in the space for distance to B.

TABLE 3.—SIGNS OF OPERATION FOR COORDINATE COMPUTATION

Bearing	Quadrant I + +	II + -	III - -	IV - +
N - E + +	+ - - + - + - +	+ - - + + - + -	- + + - + - + -	- + + - - + - +
N - W + -	+ - + - + - - +	+ - + - - + + -	- + - + - + + -	- + - + + - - +
S - W - -	- + + - + - + -	- + + - - + - +	+ - - + - + - +	+ - - + + - + -
S - E - +	- + - + - + + -	- + - + + - - +	+ - + - + - - +	+ - + - - + + -

4. Compute $\cos A = \frac{\Delta Y}{d}$ and $\sin A = \frac{\Delta X}{d}$. Enter them in Cols. 5-6 and 7-8.
5. To determine differences of distances (Cols. 5 and 6): subtract each distance from the following distance, having regard to the signs (difference $a_1 - 0$ will be entered in line P_1; $a_2 - a_1$ in line P_2; . . . finally: $d - a_n$ in line B). Enter positive differences in Col. 5 and negative ones in Col. 6. The check of this operation is that the algebraic sum of all differences equals "d".
6. The signs of operation, that is, the position of the counter control, will be taken from the sign table shown as Table 3. These signs depend both on the quadrant where points A and B appear and also on the bearing of the line AB.

In the illustration Y_A and Y_B are negative, X_A and X_B are positive, indicating (Table 1) that both points lie in quadrant IV. Either by inspection or by the fact that both ΔY and ΔX are positive (see Table 2 or Table 3) the bearing is found to be N-E. For the corresponding combination of bearing and quadrant the following set of signs is indicated:

- +   + -
- +   - + .

This complete group of signs will be used in computing the coordinates of offset points, Fundamental Operation 3; however, only the first part of the group, $\begin{smallmatrix}- & +\\ - & +\end{smallmatrix}$ will be used in computing coordinates of line-points. The first pair of signs (- +) is to be entered at the top of Cols. 5 and 6, respectively; the second pair (- +), at the bottom of the same columns. The signs preprinted on the form, which were used to aid entry of the differences of distances, are to be superceded by these signs. It would be well, therefore, to note the signs of operation with red pencil. Note also that these signs of operation taken from Table 3, and entered in red, do not describe the relation of the points with respect to one another, but rather, describe the position of the counter control to be used in calculation.

7. Enter Y_A, the Y-coordinate of the starting point, into the product dial by setting it on the keyboard and transferring it (multiply by unity). Calculation of successive coordinates involves multiplication of distances by trigonometric functions, and the successive addition of these products to a coordinate of the starting point. If the calculator is to be used to best advantage, the decimal must be considered in each entry so that results will accumulate correctly in the product dial. If the decimal were not considered, it would be necessary to do all additions and subtractions manually, and the advantages of the method described here would be largely lost. Therefore, in entering Y_A into the product dial, so set it that the number of decimal places in the product dial equals the sum of decimals in the cosine and the difference of distances.

8. Clearing only the keyboard and counter dial, so that Y_A remains in the product dial, enter the constant cos A in the keyboard.

9. Set the counter control in accordance with the red sign of operation heading the column in which the first difference of distances is located.

10. Then multiply by the first difference of distances. The result in the product dial will be the Y coordinate of the first line-point, and is to be entered in Col. 9. Clear the counter dial.

11. Multiply the cos A, which is still in the keyboard, by each of the other differences of distances in order, always setting the counter control in accordance with the corresponding sign of operation. Thus, after each multiplication the product dial will contain the resulting coordinate; the counter dial, the difference of distances last used, and the keyboard, the trigonometric function being used.

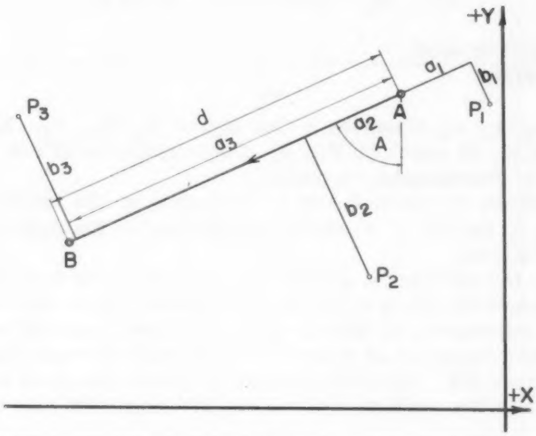
12. To check: The coordinate resulting from the final multiplication must equal Y_B.

13. The procedure for calculating the X-coordinates is similar in principle. However, sin A is used rather than cos A, and the signs of operation are taken from the bottom of Cols. 5 and 6, rather than from the top. Moreover, the computations are to be made from the bottom of the table upward so that the first machine entry will be X_B rather than X_A. The coordinate obtained from multiplication of sin A by the difference in line B will be entered in line P_4 of Col. 10; the coordinate obtained from multiplying by the difference in line P_4 will be entered in line P_3, and so forth.

14. To check: The coordinate resulting from the final multiplication must equal X_A.

The same method may be used to compute coordinates when the line is described by its bearing and the coordinates of only one point, A. Using trigonometric functions from a table, compute the coordinates of an auxiliary point, B, at some convenient distance, for example, 1,000 ft, tabulating these coordinates after those of the last line-point. Then proceed as outlined originally.

Persons more experienced with calculations of this type may prefer to rely on inspection in establishing the signs of operation, particularly as to



COORDINATES OF OFFSET POINTS								Form #2	
1	2	3	4	5	6	7	8	9	10
PT	DISTANCE	OFFSETS		$\frac{\Delta Y}{d}$ = cos A		$\frac{\Delta X}{d}$ = sin A		COORDINATES	
		RIGHT	LEFT	DIFF. OF DIST.		DIFF. OF OFFS.		Y	X
		+	−	+	−	+	−		
				0.415	0477	0.909	7998		
				⊖	⊕	⊕	⊖		
A	0	0	0					+386.31	-124.83
P_1	- 92.63		56.24		92.63		56.24	+373.59	- 17.21
P_2	129.86		185.80	222.49			129.56	+163.37	-165.86
P_3	433.23	162.48		303.37		348.28		+354.32	-586.42
B	441.13	0	0	7.90			162.48	+203.22	-526.17
				533.76	92.63	348.28	348.28	-183.09	-401.34
				-92.63					
				441.13					
				⊖	⊕	⊖	⊕		

⊕ , ⊖ INDICATE RED SIGNS

FIG. 5.

those unknown points lying between the two given points A and B. For example, in computing Y-coordinates (going from A to B), if the absolute values of the Y coordinates decrease, the sign of operation will be minus. And if moving from B to A (in computing X-coordinates), the coordinates again decrease, as in the illustration, the sign of operation will again be minus.

Fundamental Operation 3: Computation of Coordinates of Offset Points.— Fig. 5 illustrates this operation.

Given: points A and B with their coordinates Y_A, X_A, Y_B, X_B; distances a_1, a_2, a_3, measured along AB, from A; offsets b_1, b_2, b_3, measured at right angles to the line AB.

Sought: coordinates of points P_1, P_2, P_3, offset from the line AB.

The procedure is based upon the following formulae:

$$Y_P = Y_A + a \cos A - b \sin A$$

and

$$X_P = X_B - (d-a) \sin A + b \cos A$$

Form # 2 will be used.

The procedure:

1. Enter a_1, a_2, a_3 distances in Col. 2, and Y_A, X_A, Y_B, X_B coordinates in Cols. 9 and 10, as shown on Fig. 5. Compute and enter cos A, sin A and d as described in Fundamental Operation 2.

2. Enter offsets in Cols. 3 and 4, according to the following sign rule. Looking from A toward B, an offset is positive if to the right, and negative if to the left of the line.

3. Tabulate the differences of distances in Cols. 5 and 6 as in Fundamental Operation 2, algebraically subtracting each distance from the one following it. Tabulate the differences of offsets in Cols. 7 and 8 according to the same rule. Again the differences of distances can be checked since their sum should equal the distance $\overline{AB}$. If the differences of offsets have been correctly computed, the sum of the negative differences will equal the sum of the positive differences.

4. Select from Table 3 a complete set of operation signs. In the illustration, signs are selected for a line having a S-W bearing situated in Quadrant II. Enter the signs with red pencil at the top and bottom of Cols. 5, 6, 7 and 8 as shown in Fig. 5.

5. Orderly computation will be facilitated by underscoring and inserting arrows as shown in the example. Underline the 2nd, 4th, etc. (that is, even) differences of distances. Underline the 1st, 3rd, etc. (that is, odd) differences of offsets. In each line draw an arrow toward the underscored difference from the difference which is not underscored.

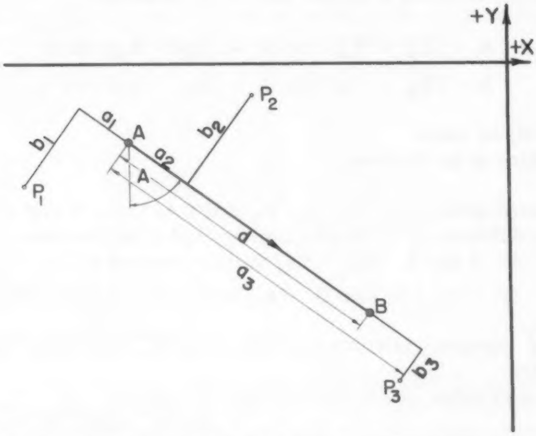
6. Set Y_A on keyboard and transfer it to the product dial, having regard to the position of the decimal. Clear keyboard and counter dial.

7. Set cos A on keyboard. Set counter control in accordance with the sign of operation above the first difference of distances. Multiply cos A by first difference of distances. Set sin A on keyboard, clear counter dial and reset counter control according to the operating sign above the first difference of offsets. Multiply sin A by the first difference of offsets. The term Y_{P_1} will then appear in product dial and should be entered in Col. 9. Again clearing the counter dial and resetting the counter control according to the sign of operation above the second difference of offsets, multiply sin A by the second difference of offsets.

The foregoing procedure may be summarized as follows: multiply in the order shown by the arrows, setting counter control in each instance according to the signs of operation entered above the multiplier. Use cos A when multiplying by differences of distances, and sin A when multiplying by differences of offsets. Read and enter answers only after multiplications by underlined differences. To check: the number appearing in the product dial after the last multiplication must be Y_B.

8. To compute the X-coordinates, clear machine. Set X_B on keyboard and transfer it to the product dial. Clear keyboard and counter dial.

9. Repeat procedure described in item 7 with the following exceptions: follow again the course indicated by the arrows but this time commence at the bottom (with the last difference of offsets in Fig. 5) and proceed toward the top. Use signs of operation prescribed under the multiplier to be used. Use cos A when multiplying by differences of offsets, and sin A when multiplying



COMPUTATION OF DISTANCES AND OFFSETS								Form # 2	
1	2	3	4	5	6	7	8	9	10
PT.	DISTANCE	OFFSETS		$\frac{\Delta Y}{d}$ = cos A		$\frac{\Delta X}{d}$ = sin A		COORDINATES	
		RIGHT	LEFT	DIFF. OF DIST.		DIFF. OF OFFS.		Y	X
		+	-	+	-	+	-		
				0.586	4751	0.809	9669		
				⊖	⊕	⊕	⊖		
A	0	0	0					- 97.41	-458.67
P_1	- 71.52	118.25			53.83		127.28	-151.24	-585.95
P_2	88.67		137.11	112.88		279.51		- 38.36	-306.44
P_3	432.56	46.76			350.61	170.70		-388.97	-135.74
B	353.57	0	0	84.20			36.55	-304.77	-172.29
				197.08	404.44	450.21	163.83	-207.36	+286.38
					+197.08	-163.83			
					-207.36	+286.38			
				⊕	⊖	⊕	⊖		

⊕ , ⊖ INDICATE RED SIGNS

FIG. 6.

by differences of distances (that is, opposite to the procedure for computing Y-coordinates). Enter answers in Col. 10 after multiplications by underlined differences. To check: the coordinate appearing in product dial after the last multiplication (that is, after multiplying by the first difference of offsets) must be X_A.

Fundamental Operation 4: Computation of Distances and Offsets.—This operation is essentially the reverse of Fundamental Operation 2 and is illustrated in Fig. 6.

Given: Points A and B, determine the line AB, and right angle offset points P_1, P_2, P_3 by their respective coordinates Y_A, X_A, Y_B, Y_{P_1}, X_{P_1}, etc.

Sought: distances a_1, a_2, a_3 and offsets b_1, b_2 and b_3.

This problem appears very frequently in field engineering whenever detail points, with known coordinates, are to be staked by means of offsets from a traverse side.

The procedure is based upon the following formulas:

$$a = (Y_P - Y_A) \cos A + (X_P - X_A) \sin A$$
$$b = (Y_B - Y_P) \sin A - (X_B - X_P) \cos A$$

Form # 2 will be used.

The procedure is as follows:

1. Enter coordinates of A, P_1, P_2, P_3 and B in Cols. 9 and 10.
2. Compute differences of Y-coordinates and tabulate them (with regard to the signs) in Cols. 5 and 6. $(Y_{P_1} - Y_A)$ will be entered in line P_1; $(Y_{P_2} - Y_{P_1})$ in line P_2, . . . $(B - Y_{P_3})$ in line B. To check: the sum of differences should equal $Y_B - Y_A$.
3. Similarly compute, tabulate in Cols. 7 and 8, and check the differences of X-coordinates.
4. Compute and enter cos A, sin A and d as before.
5. Select signs of operation from Table 4 and enter them as before at the tops and bottoms of Cols. 5, 6, 7 and 8. These signs depend on the bearing of straight line AB only, regardless of the quadrant in which the points appear.
6. Underline and arrow the differences as described for Fundamental Operation 3, preceding.
7. Set cos A on keyboard and multiply by the first difference of Y-coordinates (appearing in Col. 5 or 6). Continue to multiply in order, following the course of arrows. Use cos A with Cols. 5 and 6, and sin A with Cols. 7 and 8, setting the counter control according to the signs of operation taken from Table 4. After multiplications with underlined factors read product dial and enter answer in Col. 2 (distance). If the product dial shows a decadic complementary number, for example, 9999928.48, it indicates a negative distance, -71.52; the foot of the point is situated before A, instead of between A and B. To check: The result of the final multiplication should be "d", the distance $\overline{AB}$.

TABLE 4.—SIGNS OF OPERATION FOR OFFSET COMPUTATION

Bearing	Signs of Operation for Offset Computation
N - E + +	+ - + - + - - +
N - W + -	+ - - + - + - +
S - W - -	- + - + - + + -
S - E - +	- + + - + - + -

After having finished with the distances, clear both counter and product dials. One need not clear the keyboard since the sine or cosine which was last used will be used again. Proceed with the multiplications, commencing at the bottom, following the course of arrows, and setting the counter control according to the signs of operation at the bottom of the columns. Use cos A with Cols. 7 and 8, and sin A with Cols. 5 and 6. After multiplications with underlined differences, read offsets from product dial and enter them in Cols. 3 and 4 (offsets). Again, if the product dial shows a decadic complementary number, it indicates that the offset is negative, that is, to the left. To check: The last offset computed will be that of point A and must, consequently, equal zero.

Fundamental Operation 5: The Intersection.—This operation is illustrated in Fig. 7.

Given: Points A and B with their coordinates Y_A, X_A, Y_B and X_B. The bearing A and B of two intersecting lines.

Sought: The Y_P and X_P coordinates of point P, the point of intersection.

This is one of the most frequently used operations.

The procedure makes use of the following formulas:

$$X_C = X_A + (Y_B - Y_A) \tan A$$

$$Y_P = Y_B + \frac{X_B - X_C}{\tan A - \tan B}$$

$$X_P = X_B + (Y_P - Y_B) \tan B$$

where C is an auxiliary point.

Form # 1 will be used.

The procedure is as follows:

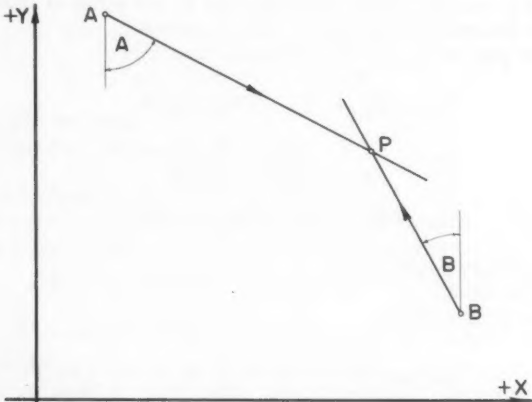
1. Enter Y_A, X_A, Y_B, X_B, tan A and tan B in form # 1 as shown in Fig. 7. The signs of the tangents must be shown. The signs depend on direction of bearing and may be obtained from Table 2. In the illustration bearing A is S-E and the sign of tan A is negative. Bearing B is N-W and sign of tan B is also negative.
2. There are three signs of operation to be obtained. The first, entered in red to the right of tan A is the sign of tan A. The second is the opposite of the sign of tan B. The third corresponds to the sign of the result of algebraically subtracting tan B from tan A.
3. Set X_A on keyboard and transfer it to the product dial. Clear keyboard and counter dial.
4. Introduce Y_A in counter dial using "+" key.
5. Set tan A on keyboard.
6. Change counter dial from Y_A to Y_B using "+" and "-" keys as required.[2]
7. Set (tan A - tan B) on keyboard.
8. Change product dial to X_B using "+" and "-" keys.[2]
9. Read answer Y_P on counter dial; enter it in the form.
10. Set tan B on keyboard.
11. Change counter dial from Y_P to Y_B using "+" and "-" keys.[2]
12. Read answer X_P on product dial; enter it in the form.

The counter and product dials must not be cleared during the procedure outlined.

[2] Note: set counter control in negative when a tangent, bearing a negative sign of operation, has been set on the keyboard. Appearance of decadic complements as answers indicates that the point falls in another quadrant.

There is no direct check in this case. Yet it is easy to verify the accuracy of the result. From the coordinates of A and P the bearing of line AP may be computed (Fundamental Operation 1) and compared to the bearing A as given. Similarly, bearing of line BP may be compared to bearing B as given.

Fundamental Operation 6: The Three-Point Problem.—Figs. 8 and 9 illustrate this operation.



INTERSECTION							Form #1
Pt.		Y		X	tan		Bearing
A	+	473.48	+	87.36	- 1.907 9770	⊖	S 62° 20' 25" E
B	+	106.17	+	514.28	- 0.528 5916	⊕	N 27° 51' 38" W
P	+	304.74	+	409.32	- 1.379 3854	⊖	
CHECK:							
A	+	473.48	+	87.36	1.908 0242		
P	+	304.74	+	409.32	(2" off)		
P-A	-	168.74	+	321.96			
B	+	106.17	+	514.28	0.528 5793		
P	+	304.74	+	409.32	(2" off)		
P-B	+	198.57	-	104.96			

⊕ , ⊖ INDICATE RED SIGNS

FIG. 7.

Given: Points A, B, and C with their coordinates; angles α and β measured between points A, B and C from the unknown position P.

Sought: Coordinates of point P: Y_P and X_P.

The answer will be determined by use of imaginary points U and V as shown in Fig. 9.

The procedure utilizes the following formulas:

$$Y_U = Y_A + (X_A - X_C) \cot \alpha$$

$$X_U = X_A - (Y_A - Y_C) \cot \alpha$$
$$Y_V = Y_B + (X_C - X_B) \cot \beta$$
$$X_V = X_B - (Y_C - Y_B) \cot \beta$$
$$\overline{CP}^2 = (Y_U - Y_V)(X_C - X_U) - (X_U - X_V)(Y_C - Y_U)$$
$$\overline{UV}^2 = (Y_U - Y_V)^2 + (X_U - X_V)^2$$
$$q = \frac{\overline{CP}^2}{\overline{UV}^2}$$
$$Y_P = Y_C + (X_U - X_V)\, q$$
$$X_P = X_C - (Y_U - Y_V)\, q$$

The procedure is as follows:

1. Enter Y_A, X_A, Y_B, X_B, Y_C, X_C, cot α, cot β in form # 1, as shown in Fig. 8. Determine and enter the coordinate differences $(Y_A - Y_C)$, $(X_A - X_C)$, $(Y_C - Y_B)$ and $(X_C - X_B)$.

2. Signs of operation may be determined as follows for each of the coordinate differences (A - C) and (C - B): For Y-coordinate differences, the sign of operation is opposite to the sign of the difference. For X-coordinate differences, the sign of operation is the same as the sign of the difference. These signs of operation are correct only when positive quantities are to be multiplied by the coordinate differences. In the procedure, cot α and cot β will be used as the multiplicands, and either or both could conceivably be negative (as when α or $\beta > 90°$). Whenever multiplication of a negative cotangent is indicated, the sign of operation will be the reverse of that described initially.

3. Set Y_A on the keyboard and transfer it to the product dial. Set cot α on the keyboard, clear counter dial. After setting counter control in accordance with the sign of operation corresponding to $(X_A - X_C)$, multiply by $(X_A - X_C)$. Read Y_U on the product dial and enter it in the form. A decadic complement indicates a negative Y_U (that is, point U falls in another quadrant). Clear both dials and the keyboard. (Certain types of calculators are supplied with a row of knobs, one above each window of the product dial. By means of these knobs a number can be introduced directly in the product dial. Whenever the procedure instructs clearly a number off the keyboard, entering another one in the product dial, and then resetting the cleared number on the keyboard: the operations of clearing, setting on the keyboard, multiplying by the unit and resetting the original number, can be avoided by using the above mentioned set of knobs.)

4. Enter X_A in the product dial, cot α on the keyboard. Multiply by $(Y_A - Y_C)$, using the proper position of the counter control. Read X_U on the product dial and enter it in the form. Clear both dials and the keyboard.

5. Enter Y_B in the product dial, cot β on the keyboard. Multiply by $(X_C - X_B)$. Read Y_V on the product dial and enter it in the form. Clear both dials and the keyboard.

6. Set X_B in the product dial, cot β on the keyboard. Multiply by $(Y_C - Y_B)$. Read X_V on product dial and enter it in the form. Clear both dials and the keyboard.

7. Compute $(Y_U - Y_V)$ and $(X_U - X_V)$, and enter them in the form. Set $(Y_U - Y_V)$ on the keyboard. Multiply it by $(X_C - X_U)$, after having set the

counter control properly: in positive when the two coordinate differences have identical signs, and in negative otherwise. Clear keyboard and counter dial but do not clear the product dial. Set $(X_U - X_V)$ on keyboard. Multiply by $(Y_C - Y_U)$. Since the product of the latter two differences is to be subtracted from the quantity in the product dial, the counter control must be set in positive when the two differences have different signs, and in negative, if the signs of the two differences are identical. Read $\overline{CP}^2$ on the product dial. When en-

THREE POINT PROBLEM						Form #1
Pt.		Y		X		
A	+	652.34	+	72.86	cot α : + 1.359 3312	α = 36° 20' 25"
C	+	854.29	+	564.19	cot β : + 0.451 1088	β = 65° 43' 10"
A-C	-	201.95	-	491.33		
		⊕		⊖		
B	+	545.03	+	1272.44		
C	+	854.29	+	564.19		
C-B	+	309.26	-	708.25		
		⊖		⊖		
U	-	15.54	+	347.38	CP^2 = + 631028.57	
V	+	225.53	+	1132.93		
U-V	-	241.07	-	785.55	UV^2 = + 675203.55	
		⊕		⊖		
C	+	854.29	+	564.19		
U	-	15.54	+	347.38	$q = \frac{CP^2}{UV^2}$ = + 0.934 5753	
C-U	+	869.83	+	216.81		
P	+	120.13	+	789.49		
		C H E C K :				
P	+	120.13	+	789.49	tan: 1.346 5174	N 53° 24' 01" W
A	+	652.34	+	72.86		- 17° 03' 37"
A-P	+	532.21	-	716.63		α = 36° 20' 24"
P	+	120.13	+	789.49	tan: 0.306 8813	N 17° 03' 37" W
C	+	854.29	+	564.19		+ 48° 39' 31"
C-P	+	734.16	-	225.30		β = 65° 43' 08"
P	+	120.13	+	789.49	tan: 1.136 6204	N 48° 39' 31" E
B	+	545.03	+	1272.44		
B-P	+	424.90	+	482.95		

⊕ , ⊖ INDICATE RED SIGNS

FIG. 8.

tering it in the form, show the sign of the result too: a decadic complement indicates that $\overline{CP}^2$ is negative. Clear the dials and the keyboard.

8. To compute the square of $\overline{UV}$ distance, set $(Y_U - Y_V)$ on keyboard and multiply it by itself, using the counter control in positive. Then clear counter dial and the keyboard, but do not clear the product dial. Square $(X_U - X_V)$, with the counter control again set in positive. Read $\overline{UV}^2$ on the product dial and enter it in the form. It is always positive. Clear both dials and the keyboard.

9. Compute $q = \overline{CP}^2/\overline{UV}^2$. Enter it in the form. Note with red the signs of

operation corresponding to the (U - V) coordinate differences. The signs of operation follow the basic rule set out in item 2, with the qualification that if q is negative then the signs will be reversed just as they were when cot α or cot β was negative.

10. Enter Y_C in the product dial and set q on the keyboard. Multiply by $(X_U - X_V)$, again setting the counter control in accordance with the signs of operation corresponding to $(X_U - X_V)$. Read Y_P on product dial and enter it in the form. Clear both dials and the keyboard.

11. Enter X_C in the product dial, set q in the keyboard. Multiply by $(Y_U - Y_V)$, using the proper position of the counter control. Read X_P on the product dial, and enter it in the form. Clear the machine.

12. The best way of checking the above computation is to compute bearings of lines PA, PB and PC, using the coordinates of points A, B, C and P. The difference of the proper bearings must equal α and β, respectively.

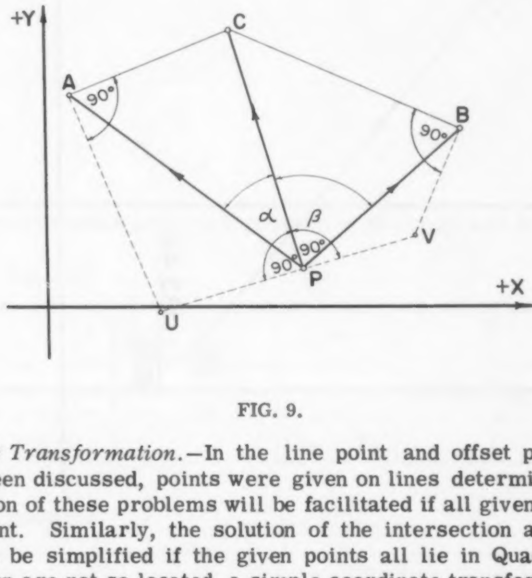


FIG. 9.

Coordinate Transformation.—In the line point and offset point problems which have been discussed, points were given on lines determined by coordinates. Solution of these problems will be facilitated if all given points lie in a single quadrant. Similarly, the solution of the intersection and three-point problems can be simplified if the given points all lie in Quadrant I. If the points as given are not so located, a simple coordinate transformation may be performed. For example, to operate with straight line AB in Fig. 10, we perform the transformation by shifting the X-axis south by a convenient value which is larger than Y_B. The dashed line shows the temporary, transferred X-axis. Now the entire straight line AB falls in the same quadrant, (II). If the original coordinates were

Y_A:	+ 304.17	X_A:	- 511.75
Y_B:	- 183.49	X_B :	- 116.16

then after the transfer of the X axis by 200 ft the following coordinates are obtained in the temporary, transformed system.

Y_A: + 504.17 X_A: - 511.75

Y_B: + 16.51 X_B: - 116.16

Now points A and B bear uniform signs indicating that they are in the same quadrant.

After completing the operation, retransform the points by simply subtracting 200 from each Y-coordinate.

To solve an intersection or a three-point problem, it is desirable to bring all given points to quadrant I. This can be done as in Fig. 10, except, that in some cases it will be necessary to move both axes.

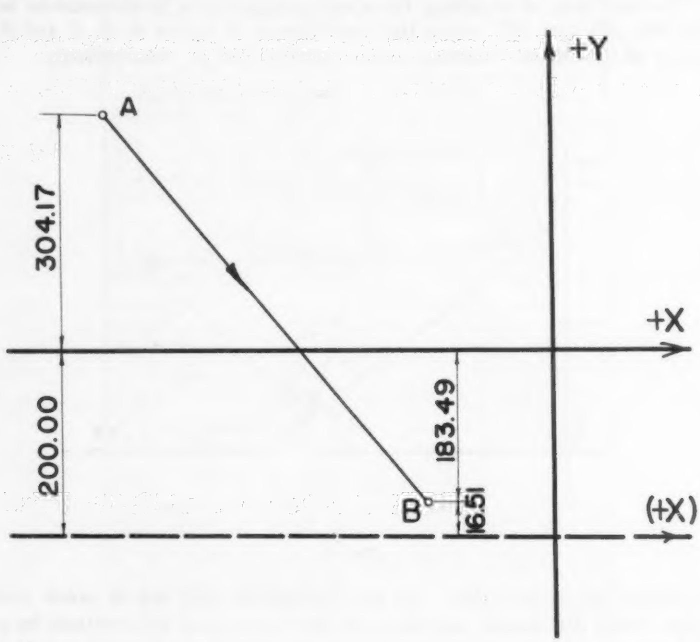


FIG. 10.

If the points are in quadrant III, the simplest transformation is to change the negative signs to positive (that is, to rotate the coordinate system 180°) and to change again the signs of the answer after the computation.

Operation Diagrams.—In the following a summary of the fundamental operations will be presented, in form of the "Operation Diagrams."

The diagrams demonstrate each phase of the operations. The order of the operation is from left to right, each column representing a step.

The following notation will be used.

P : Product dial
C : Counter dial
K : Keyboard

/ indicates: "clear the device opposite which this sign appears."
:-: indicates auxiliary numbers which appear in one of the dials but need not be read or written down.
Underlined expressions indicate answers to be read and entered in the form.

Expressions not underlined indicate numbers to be introduced into a dial or set on the keyboard. When an expression appears for entering into a dial already containing another quantity, it indicates that the number already in the dial should be altered by use of the plus or minus key.

In entering each quantity, there must be borne in mind both the sign of the quantity itself and the sign of operation. When a quantity in the diagram is preceded by a negative sign, it indicates that the sign of operation is opposite to the algebraic sign of the quantity. In Fundamental Operations 2, 3 and 4, however, no indication of signs of operation has been attempted because these signs presumably will have been determined by reference to the tables as discussed therein.

Fundamental Operation 1: Computation of Bearings and Distances

a. Bearing

P			$(X_B - X_A)$					/
C		1		/		DIVIDE	$\underline{\tan A}$	/
K	$(X_B - X_A)$			/	$(Y_B - Y_A)$			/

b. Distance

P			:-:				$\underline{d^2}$	/
C		$(Y_B - Y_A)$		/		$(X_B - X_A)$		/
K	$(Y_B - Y_A)$			/	$(X_B - X_A)$			/

Fundamental Operation 2: Computation of Coordinates of Line-Points

P			Y_A				$\underline{Y_{P_1}}$			$\underline{Y_{P_2}}$			$\underline{Y_{P_3}}$	
C		1		/		a_1		/	$(a_2 - a_1)$		/	$(a_3 - a_2)$		/
K	Y_A			/	cos A									

P	. . .		$\underline{Y_{P_n}}$			* $\underline{Y_B}$	/			X_B		
C	. . .	$(a_n - a_{n-1})$		/	$(d - a_n)$		/		1		/	
K	. . .						/	X_B			/	sin A

P		$\underline{X_{P_n}}$			$\underline{X_{P_{n-1}}}$		. . .		$\underline{X_{P_1}}$			* $\underline{X_A}$	/
C	$(d - a_n)$		/	$(a_n - a_{n-1})$		/	. . .	$(a_2 - a_1)$		/	a_1		/
K							. . .						/

Fundamental Operation 3: Computation of Coordinates of Offset Points

P			Y_A				:-:				$\underline{Y_{P_1}}$			:-:		
C		1		/		a_1		/		b_1		/	$(b_2 - b_1)$		/	
K	Y_A			/	cos A			/	sin A						/	cos A

P		$\underline{Y_{P_2}}$		. . .		:-:				$\underline{Y_{P_n}}$		
C	$(a_2 - a_1)$		/	. . .	$(a_n - a_{n-1})$		/		$(b_n - b_{n-1})$		/	b_n
K				. . .			/	sin A				

P	:-:				* $\underline{Y_B}$	/			X_B		**		:-:		
C		/		$(d - a_n)$		/		1		/		b_n		/	
K		/	cos A			/	X_B			/	cos A			/	sin A

*To be checked.

**If there is an odd number of offset points, as in Fig. 5 and 6. If there is an even number of them, sin A will be entered here instead, as the arrows show.

P		$\underline{X_{P_n}}$			:-:				$\underline{X_{P_{n-1}}}$		. . .
C	$(d - a_n)$		/	$(a_n - a_{n-1})$		/		$(b_n - b_{n-1})$		/	. . .
K						/	cos A				. . .

P		:-:				$\underline{X_{P_1}}$			:-:				*$\underline{X_A}$	/
C	$(b_2 - b_1)$		/		$(a_2 - a_1)$		/	a_1		/		b_1		/
K			/	sin A						/	cos A			/

Fundamental Operation 4: Computation of Distances and Offsets

P			:-:				$\underline{a_1}$			:-:		
C		$(Y_{P_1} - Y_A)$		/		$(X_{P_1} - X_A)$		/	$(X_{P_2} - X_{P_1})$		/	
K	cos A			/	sin A						/	cos A

P		$\underline{a_2}$		. . .		:-:				$\underline{a_n}$
C	$(Y_{P_2} - Y_{P_1})$		/	. . .	$(Y_{P_n} - Y_{P_{n-1}})$		/		$(X_{P_n} - X_{P_{n-1}})$	/
K				. . .			/	sin A		

P		:-:				*$\underline{d}$	/		:-:				$\underline{b_n}$
C	$(X_B - X_{P_n})$		/		$(Y_B - Y_{P_n})$		/	$(X_B - X_{P_n})$		/		$(Y_B - Y_{P_n})$	
K			/	cos A						/	sin A		

P			:-:				$\underline{b_{n-1}}$		. . .		:-:	
C	/	$(Y_{P_n} - Y_{P_{n-1}})$		/		$(X_{P_n} - X_{P_{n-1}})$		/	. . .	$(X_{P_2} - X_{P_1})$		/
K				/	cos A				. . .			/

P			$\underline{b_1}$			:-:				*0	/
C		$(Y_{P_2} - Y_{P_1})$		/	$(Y_{P_1} - Y_A)$		/		$(X_{P_1} - X_A)$		/
K	sin A						/	cos A			/

*To be checked.

Fundamental Operation 5: Intersection

P		X_A					:-:			X_B					$\underline{X_P}$	/
C	1		/	Y_A		Y_B					$\underline{Y_P}$			Y_B		/
K X_A			/		tan A			/	(tan A - tan B)			/	-tan B			/

Fundamental Operation 6: The Three-Point Problem

P			Y_A				$\underline{Y_U}$	/			X_A		
C		1		/		$(X_A - X_C)$		/		1		/	
K	Y_A			/	cot α			/	X_A			/	cot α

P		$\underline{X_U}$	/			Y_B				Y_V	/			X_B
C	$-(Y_A - Y_C)$		/		1		/		$(X_C - X_B)$		/		1	
K			/	Y_B			/	cot β			/	X_B		

P				$\underline{X_V}$	/			:-:		
C	/		$-(Y_C - Y_B)$		/		$(X_C - X_U)$		/	
K	/	cot β			/	$(Y_U - Y_V)$			/	$(X_U - X_V)$

P		$\underline{\overline{CP}^2}$	/			:-:			
C	$-(Y_C - Y_U)$		/		$(Y_U - Y_V)$		/		$(X_U - X_V)$
K			/	$(Y_U - Y_V)$			/	$(X_U - X_V)$	

P	$\underline{\overline{UV}^2}$	/			$\overline{CP}^2$				/			Y_C			
C		/		1		/		DIVIDE $\underline{q}$	/		1		/		$(X_U - X_V)$
K		/	$\overline{CP}^2$			/	$\overline{UV}^2$		/	Y_C			/	q	

P	$\underline{Y_P}$	/			X_C				$\underline{X_P}$	/
C		/		1		/		$-(Y_U - Y_V)$		/
K		/	X_C			/	q			/

In the foregoing operations, the value of a sketch of the problem should not be overlooked. While the sketch need not be scale-correct, the approximate proportion of the distances and the general directions of the bearings should correspond with the data of the actual problem. This procedure will preclude sign-errors, erroneous part answers and the carrying forward of such errors. Moreover, the visual check provides a safeguard against answers which are actually reflected images of the correct values.

APPLICATIONS

Several recurrent applications will be discussed in the following. Their solution can be considerably simplified by using one or more of the Fundamental Operations. The procedure will be described very briefly. It is acknowledged that these examples represent only a small portion of the possible applications. Space limitations preclude the showing of more cases where this method could be used advantageously.

Coordinates of the Center of a Circular Curve.—This problem is illustrated in Fig. 11.

Given: Y_A, X_A, Y_B, X_B, Y_C, X_C coordinates of three circumferential points (A, B, C).

Sought: Y_o and X_o coordinates of center O, and r, the radius.

The procedure:

1. Compute coordinates of auxiliary points D and E.

$$Y_D = \frac{Y_A + Y_B}{2}; \quad X_D = \frac{X_A + X_B}{2}; \quad Y_E = \frac{Y_B + Y_C}{2}; \quad X_E = \frac{X_B + X_C}{2}$$

2. Compute tan D and tan E:

$$\tan D = -\cot A = -\frac{Y_B - Y_A}{X_B - X_A}; \quad \tan E = -\cot C = -\frac{Y_B - Y_C}{X_B - X_C}$$

3. Intersect DO and EO straight lines, each determined by a point and a bearing. (Fundamental Operation 5, Intersection). The answer is Y_o and X_o.

4. Compute distance $\overline{AO} = r$. (Fundamental Operation 1, Computation of Bearings and Distances).

5. Check: distance $\overline{AO}$ must be equal to distance $\overline{BO}$ and $\overline{CO}$.

Intersection of a Circular Curve with a Straight Line.—See Fig. 12 illustrates this problem.

Given: The straight line AB with its coordinates Y_A, X_A, Y_B, X_B; C, the center of the curve with coordinates Y_C, X_C; r the radius.

Sought: Y_{P_1}, X_{P_1} coordinates of point of intersection, P_1.

The procedure:

1. Compute "a" distance and "b" offset of point C with respect to straight line AB. (Fundamental Operation 4, Computation of Distances and Offsets).

2. Compute x; $x^2 = r^2 - b^2$

3. Compute coordinates of point P_1 as a line point on straight line AB. Distance from A: (a - x). (Fundamental Operation 2, Coordinates of Line-

points). If P_2 is also needed, it may be obtained by using (a + x) as the distance from A.

4. Check: Distance $\overline{P_1C}$ must be equal to r.

This computation may be advantageously used for laying out a curve which is intersected by a traverse line: detail curve-points P_1 and P_2 could be

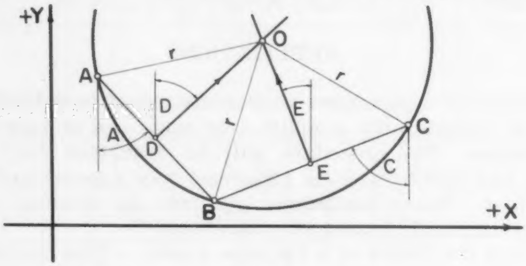


FIG. 11.

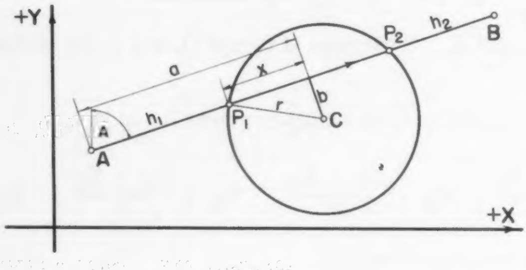


FIG. 12.

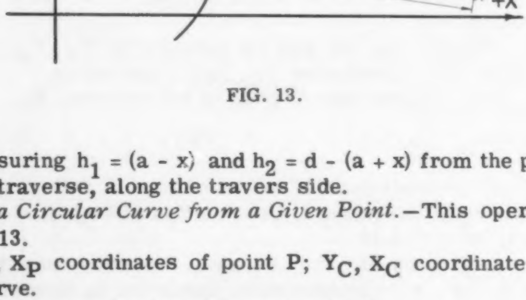


FIG. 13.

staked by measuring $h_1 = (a - x)$ and $h_2 = d - (a + x)$ from the points of intersection of the traverse, along the travers side.

Tangent to a Circular Curve from a Given Point.—This operation is illustrated in Fig. 13.

Given: Y_P, X_P coordinates of point P; Y_C, X_C coordinates of center C; r radius of curve.

Sought: Y_T, X_T coordinates of tangent point T.

The procedure:

1. Compute distance $d = \overline{CP}$, and bearing P, from the given coordinates (Fundamental Operation 1, Computation of Bearings and Distances).

2. Compute β ; $\sin \beta = r/d$

3. Compute distance f; $f^2 = d^2 - r^2$.

4. Compute coordinates of T as a line point f distance from P at bearing $(P - \beta)$. (Fundamental Operation 2, Coordinates of Line-points).

5. Check: distance $\overline{CT}$ computed from coordinates must be equal with r radius.

Alternative procedure, which avoids using trigonometric tables:

1. Compute distance d; $d = \overline{CP}$.
2. Compute distance f: $f^2 = d^2 - r^2$.
3. Compute distance x; $x = r^2/d$

4. Compute distance b; $b = r\,f/d$

5. Compute coordinates of T as an offset point, with respect to straight line PC, with distance $(d - x)$ and offset b. (Fundamental Operation 3, Coordinates of Offset Points).

6. Check: $x^2 + b^2 = r^2$; $\overline{CT} = r$; $\overline{TP} = f$.

Common Outside Tangent of Two Circular Curves.—Fig. 14 illustrates this problem.

Given: $Y_{C_1}, X_{C_1}, Y_{C_2}, X_{C_2}$ coordinates of C_1 and C_2 centers of the two curves; r_1, r_2 radii of the two curves.

Sought: $Y_{E_1}, X_{E_1}, Y_{E_2}, X_{E_2}$ coordinates of the tangent points.

The procedure:

1. Compute distance d; $d = \overline{C_1C_2}$; Compute bearing A. (Fundamental Operation 1, Computation of Bearings and Distances).

2. Compute distance a; $a = \dfrac{r_2}{r_1 - r_2}\,d$

3. Compute coordinates of point A as a line-point on straight line $C_1\,C_2$ at distance $(d + a)$ from C_1. (Fundamental Operation 2, Coordinates of Line-points).

4. Compute β; $\sin \beta = \dfrac{r_1}{(d + a)}$

5. Compute t_1 and t_2; $t_1^2 = (a + d)^2 - {r_1}^2$; ${t_2}^2 = a^2 - {r_2}^2$

6. Compute coordinates of E_1 and E_2 as line-points at distances t_1 and t_2 from A on a line having a bearing $(A - \beta)$. (Fundamental Operation 2, Coordinates of Line-points).

7. Check: $\overline{E_1C_1}$ distance must equal r_1; $\overline{E_2C_2}$ must equal r_2.

Common Inside Tangent of Two Circular Curves.—This problem is illustrated in Fig. 15.

Given: $Y_{C_1}, X_{C_1}, Y_{C_2}, X_{C_2}$ coordinates of the centers of the two curves; r_1, r_2 radii of the curves.

Sought: $Y_{E_1}, X_{E_1}, Y_{E_2}, X_{E_2}$ coordinates of the tangent points.

The procedure:

1. Compute distance d; $d = \overline{C_1C_2}$; compute bearing A. (Fundamental Operation 1, Computation of Bearings and Distances).

2. Compute distance a; $a = d\ r_2/(r_1 + r_2)$

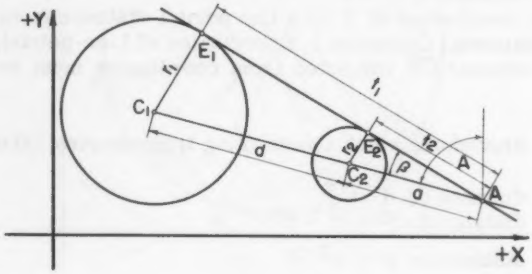


FIG. 14.

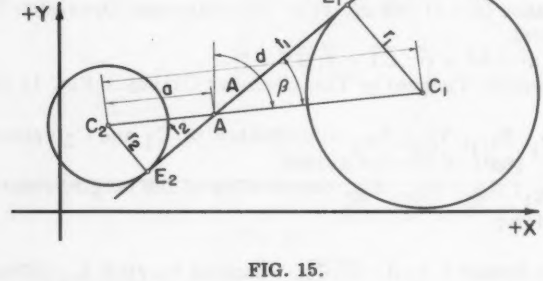


FIG. 15.

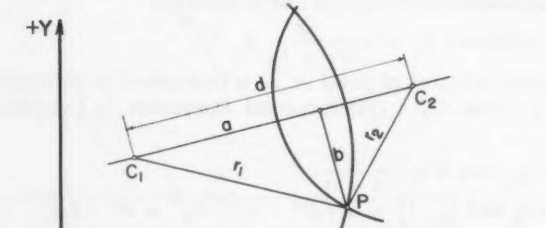


FIG. 16.

3. Compute coordinates of A as a line-point on straight line $C_1\ C_2$, at distance a from C_2. (Fundamental Operation 2, Coordinates of Line-points).

4. Compute β; $\sin \beta = r_1/(d-a)$

5. Compute distances t_1 and t_2; $t_1^{\,2} = (d-a)^2 - r_1^{\,2}$; $t_2^{\,2} = a^2 - r_2^{\,2}$

6. Compute coordinates of E_1 and E_2 as line-points on a straight line determined by point A and bearing $(A - \beta)$.

Intersection of Two Circular Curves.—Fig. 16 illustrates this operation.

Given: Y_{C_1} X_{C_1}, Y_{C_2}, X_{C_2} coordinates of the centers of the two curves; r_1, r_2 radii of the two curves.

Sought: Y_P, X_P coordinates of a point of intersection of the two curves.

The procedure:

1. Compute distance d; $d = C_1C_2$ (Fundamental Operation 1, Computation of Bearings and Distances).

2. Compute a; $a = r_1^2 - r_2^2 + d^2/(2\ d)$

3. Compute b; $b^2 = r_1^2 - a^2$

4. Compute point P as an offset point of straight line C_1 C_2, with distance a from C_1, and offset b.

5. Check: distance $\overline{C_1P} = r_1$; distance $\overline{C_2P} = r_2$.

Circular Curve Joining Two Given Circular Curves.—In Fig. 17.

Given: Y_{C_1}, X_{C_1}, Y_{C_2}, X_{C_2} coordinates of the two curve-centers; r_1, r_2 radii of the two curves; r radius of the common tangent curve.

Sought: Coordinates of point C (center of the common tangent curve) and coordinates of tangent points E_1, E_2.

The procedure:

The center of the common tangent curve is simply the point of intersection of two curves with centers C_1 and C_2 and radii $(r - r_1)$ and $(r - r_2)$, respectively.

1. Compute C as the point of intersection of the above described two curves, according to the procedure outlined in paragraph "f".

2. Compute the coordinates of E_1 and E_2 as line-points at distance r from C on lines CC_1 and CC_2, respectively.

3. Check: distance $\overline{E_1C_1} = r_1$; distance $\overline{E_2C_2} = r_2$.

Curve Joining Another Curve and a Tangent.—In Fig. 18:

Given: coordinates of points A and B determining a straight line; coordinates of C_1, center of a given curve; r_1 the radius of the given curve; r the radius of the desired common tangent curve.

Sought: coordinates of C center of the common tangent curve; coordinates of tangent points E_1 and E_2.

The procedure:

1. Establish point D as any point on a straight line which is parallel to line AB at r distance from it. The simplest approach is the computation of coordinates of D as a line point on a straight line determined by point A and a bearing perpendicular to that of line AB. To compute coordinates along a line at right angles to a line of known bearing, transpose the sin and cos of the known bearing in accordance with the formulae $\sin(90° + A) = \cos A$ and $\cos(90° + A) = -\sin A$.

2. Compute the distance a and offset b of point C_1 referred to a straight line determined by point D and the bearing of line AB. (Fundamental Operation 4, Computation of Distances and Offsets).

3. Compute distance x; $x^2 = (r - r_1)^2 - b^2$

4. Establish point C as a line-point on straight line CD at distance (a - x) from D. (Fundamental Operation 2, Coordinates of Line-points.)

5. Compute coordinates of E_1, on line AB at distance (a - x) from A.
6. Compute E_2 on line CC_1 at distance r from C.
7. Check: distance $\overline{CE_1} = r$; distance $\overline{C_1E_2} = r_1$.

Minimum Distance Between Two Circular Curves.—This problem is illustrated in Fig. 19.

Given: Coordinates of C_1 and C_2 centers of the curves and radii r_1 and r_2.
Sought: x, minimum distance; E_1, E_2 nearest points on the two curves.

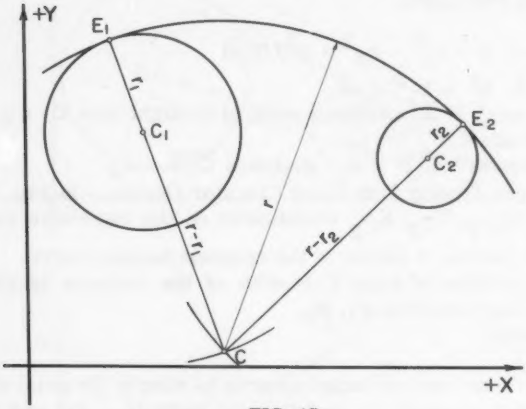


FIG. 17.

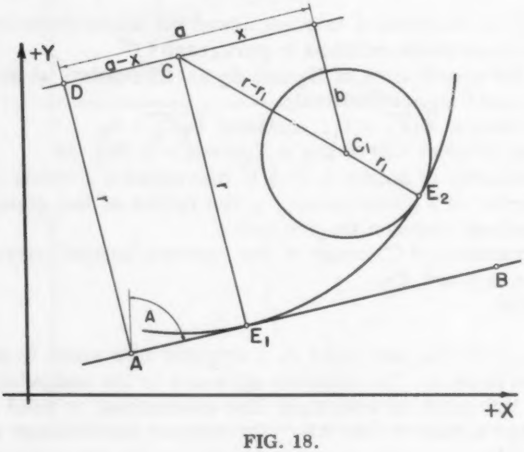


FIG. 18.

The procedure:

1. Compute distance d; $d = C_1C_2$ (Fundamental Operation 1, Computation of Bearings and Distances).
2. Compute distance x; $x = (r_1 - r_2 - d)$
3. Compute E_1 and E_2 on line C_1C_2 at distances r_1 and $(d + r_2)$, respectively, from C_1.

4. Check: $E_1E_2 = x$.

A Circular Curve Joining a Tangent at a Given Point and Passing Through a Given Point.—In Fig. 20:

Given: Coordinates of points A and B (determining the tangent), where A is the desired beginning of curve; coordinates of point P, to be passed through

Sought: Coordinates of point C center of the curve and r, its radius.

The procedure:

1. Compute bearing A of straight line AB (Fundamental Operation 1, Computation of Bearings and Distances).
2. Compute distance $d = \overline{AP}$ and bearing P of line AP (Fundamental Operation 1, Computation of Bearings and Distances).
3. $\beta = (A - P)$
4. $r = \dfrac{d}{2 \sin \beta}$
5. Compute coordinates of point C as a line point on a line determined by point A and perpendicular to AB. Distance from A is r. (Fundamental Operation 2, Coordinates of Line-points).
6. Check: Distance $\overline{CP} = r$.

Computation of Reverse Curves Between Diverging Tangents with Given Beginning of Curve.—

Without spirals (Fig. 21).—

Given: coordinates of points A, B, D, E, determining the two tangents to be connected. A is required to be the beginning of the reverse curve; r_1 and r_2 the specified radii of the curves.

Sought: Centers C_1, C_2 and central angles α_1, α_2.

The procedure:

1. Compute bearings A and D (Fundamental Operation 1, Computation of Bearings and Distances); compute β; $\beta = (A - D)$.
2. Compute C_1 as a line point on a line determined by point A and perpendicular to line AB; distance from A: r_1 (Fundamental Operation 2, Coordinates of Line-points).
3. Establish point F on a line determined by any point of line DE and perpendicular to DE, at distance r_2. (It is convenient to choose D as the foot). (Fundamental Operation 2, Coordinates of Line-points).
4. Compute distance a and offset b of point C_1 referred to a straight line determined by point F and bearing D (Fundamental Operation 4, Computation of Distances and Offsets).
5. Compute distance c; $c^2 = (r_1 + r_2)^2 - b^2$.
6. Compute coordinates of point C_2 on a line determined by point F and bearing D at distance (a + c) from point F (Fundamental Operation 2, Coordinates of Line-points).
7. Compute α_1 and α_2; $\tan \alpha_2 = c/b; \alpha_1 = \alpha_2 + \beta$
8. Check: distance $\overline{C_1C_2} = (r_1 + r_2)$.

With Spirals (Fig. 22).—

Given: coordinates of points A, B, D, E, determining the two tangents to be connected. A is required to be the beginning of the spiral; r_1 and r_2 the specified radii of the two curves; x_1, x_2 distances and o_1, o_2 offsets of those imaginary points of the circular curves produced where the tangents of the curves are parallel to the given tangents.

Sought: Coordinates of C_1, C_2 centroids, and α_1, α_2 central angles of the circular cuves.

The procedure:

1. Compute bearings A and D (Fundamental Operation 1, Computation of Bearings and Distances). Compute β; β = (A - D).

2. Compute C_1 as an offset point with respect to line AB having distance x_1 and offset $(r_1 + o_1)$. (Fundamental Operation 3: Coordinates of Offset Points.)

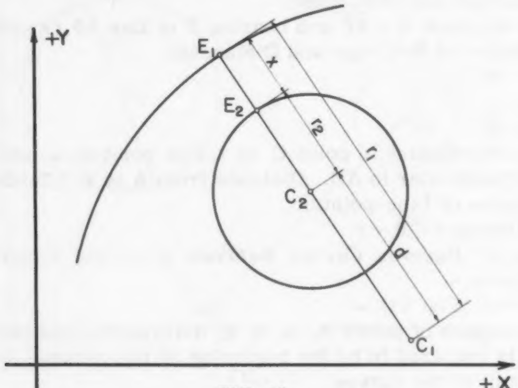


FIG. 19.

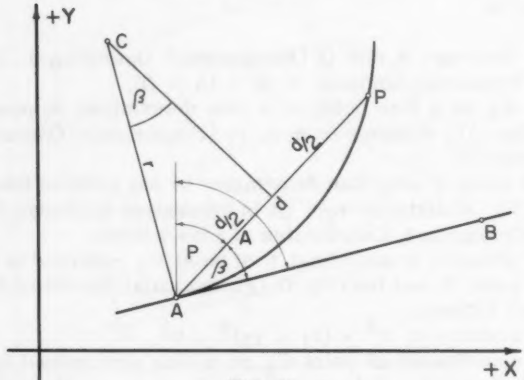


FIG. 20.

3. Establish point F as a line-point on a line determined by any point of line DE, perpendicular to line DE. The distance from F is $(r_2 + o_2)$. (It is convenient to choose D as the foot.) (Fundamental Operation 2: Coordinates of Line-points.)

4. Compute distance a and offset b of point C_1 referred to the line determined by point F and bearing D. (Fundamental Operation 4: Computation of Distances and Offsets).

5. Compute distances $\overline{C_1C_2}$ = f (From triangle C_1C_2G)

$$f^2 = (r_1 + o_1 + r_2 + o_2)^2 + (x_1 + x_2)^2$$

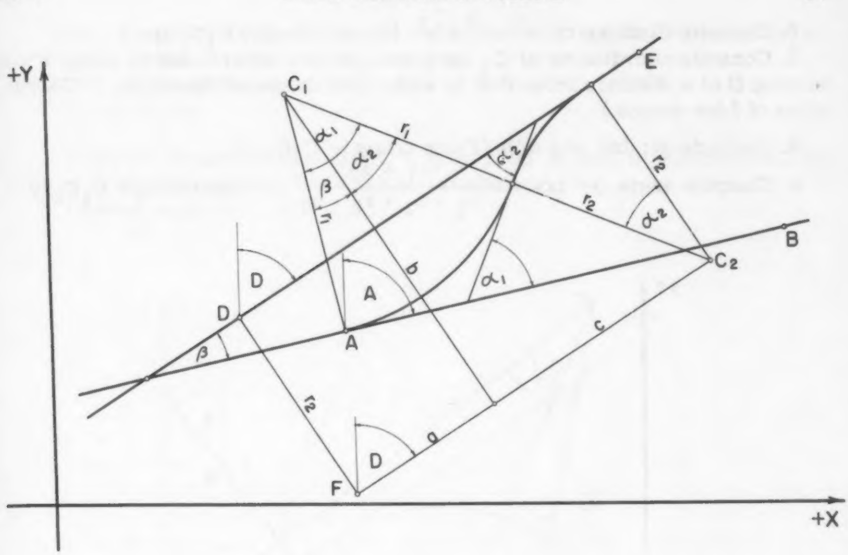

+Y
+X
C1
C2
E
B
D
A
F
α1
α2
β
r1
r2
b
c
a
D
A

FIG. 21.

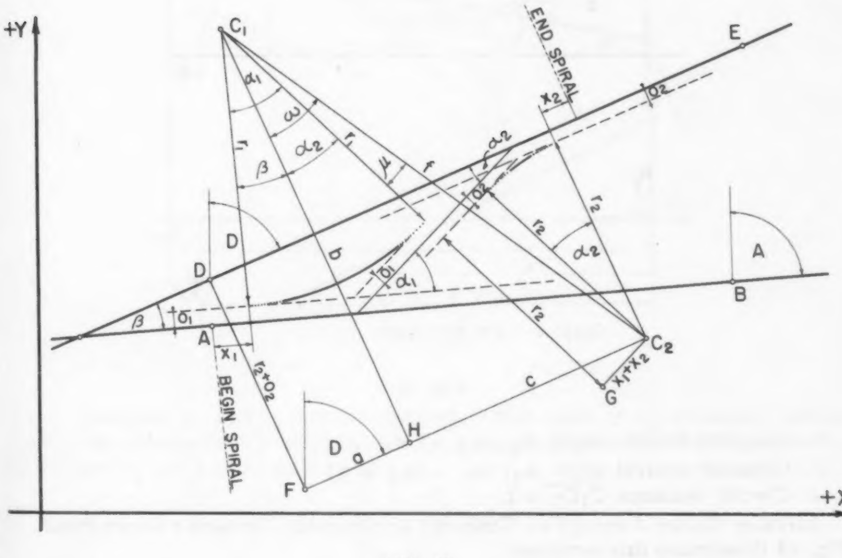

+Y
+X
C1
C2
E
B
D
A
F
G
H
END SPIRAL
BEGIN SPIRAL
α1
α2
ω
β
μ
f
r1
r2
r2+o2
x1
x2
x1+x2
o1
o2
b
c
a
D
A

FIG. 22.

6. Compute distance c; $c^2 = f^2 - b^2$ (From triangle C_1C_2H)

7. Compute coordinates of C_2 on a straight line determined by point F and bearing D at a distance from F of (a + c). (Fundamental Operation 2: Coordinates of Line-points.)

8. Compute ω; $\tan \omega = c/b$ (From triangle $C_1C_\omega H$)

9. Compute angle μ; $\tan \mu = \dfrac{(x_1 + x_2)}{r_1 + o_1 + r_2 + o_2}$ (From triangle C_1C_2G).

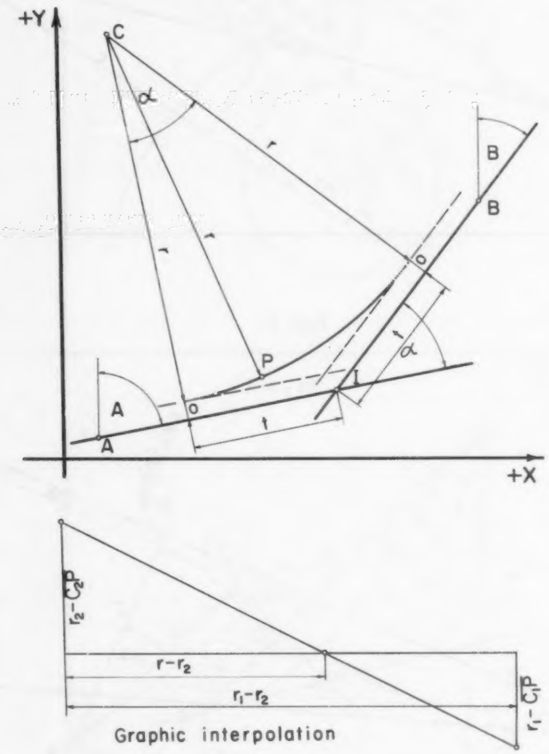


FIG. 23.

10. Compute central angle α_2; $\alpha_2 = (\omega - \mu)$

11. Compute central angle α_1: $\alpha_1 = (\alpha_2 + \beta)$

12. Check: distance $\overline{C_1C_2} = f$.

Circular Curve Joining Two Tangents and Passing Through a Given Point.— Fig. 23 illustrates this problem.

There are several solutions of this problem. The one illustrated is applicable to curves with spirals as well as to simple curves.

The solution can not be reached directly if the curve is to have spirals: trial solutions are required.

Given: Coordinates of points A, I and B, determining the two tangents; coordinates of point P to be passed through.

Sought: r the radius and C the center of the curve.

The procedure:

1. Compute distances $\overline{AI}$ and $\overline{BI}$, and bearings A and B (Fundamental Operation 1: Computation of Bearings and Distances).

2. Central angle $\alpha = (A - B)$.

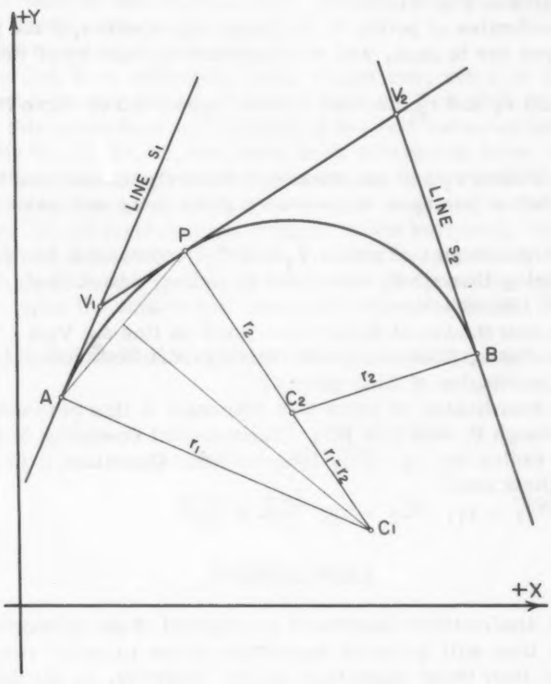


FIG. 24.

3. Compute o_1 and t_1 for the trial r_1. The value of o_1 is usually determined by reference to a spiral table; t_1 is the tangent length of a curve having a radius $(r_1 + o_1)$ and central angle α.

$$t_1 = (r_1 + o_1) \tan 1/2\ \alpha$$

4. Compute coordinates of point C_1 center of the trial curve, as an offset point referred to line IA at distance t_1 and offset $(r_1 + o_1)$, and also, as a check, recompute the coordinates of C_1 using the same distance and offset referred to line IB. (Fundamental Operation 3: Coordinates of Offset Points.)

5. Compute distance $\overline{C_1P}$. (Fundamental Operation 1, Computation of Bearings and Distances.) If it is shorter, than r_1, that means the radius must be increased, and if it is longer, the radius must be decreased.

6. Repeat the procedure with a new, corrected radius (r_2).

7. If the first two trial radii were close enough to the required radius, a large scale graphic interpolation showing $(r_1 - r_2)$ on one axis, and $(r_1 - C_1P)$ and $(r_2 - C_2P)$ on the other axis will usually give the final, correct radius, which should be checked by repeating the procedure.

Compound Curves, Beginning at a Given Point of a Straight Line, Passing Through Another Given Point, and Joining Another Straight Line.—This problem is illustrated in Fig. 24.

Given: Coordinates of points A the beginning of curve, P the point through which the curves are to pass, and straight lines s_1 and s_2 of known orientation.

Sought: radii r_1 and r_2, centers C_1 and C_2, and end of curve B.

The procedure:

1. Compute radius r_1 and coordinates of point C_1, as outlined in application j (A circular curve joining a tangent at a given point and passing through a given point).

2. Compute coordinates of points V_1 and V_2: intersect a line perpendicular to C_1P and passing through P, with lines s_1 and s_2, respectively. (Fundamental Operation 5: Intersection.)

3. Compute coordinates of B, as a line point on line s_2; $V_2B = V_2P$. (Fundamental Operation 1, Computation of Bearings and Distances;) (Fundamental Operation 2, Coordinates of Line-points.)

4. Compute coordinates of point C_2: intersect a line perpendicular to s_2 and passing through B, with line PC_1. (Fundamental Operation 5: Intersection.)

5. Compute radius r_2; $r_2 = \overline{BC_2}$ (Fundamental Operation 1: Computation of Bearings and Distances.)

6. Check: $\overline{PC_1} = r_1$; $\overline{PC_2} = r_2$; $\overline{V_1A} = \overline{V_1P}$.

CONCLUSIONS

The several applications illustrated are typical of the methods of solution. It is hoped that they will prove of immediate value to many readers. Even more important than their immediate utility, however, is the fact that they demonstrate two requisites for getting the most from the desk calculator, mastery of the fundamental operations and practice in analysis of everyday problems in terms of those operations. For such problems, and confessedly, for the problems illustrated, the reader can and possibly may prefer to develop his own procedure for using these fundamental operations. Their use however will materially shorten the time for analysis and contribute significantly to the accuracy of the results.

ACKNOWLEDGEMENTS

The author, having spoken English for but a short while, is indebted to Mr. Raymond O'Neil[3] for his criticism and revision of the text.

[3] Highway Engineer, Tudor Eng. Co., San Francisco, Calif.

APPENDIX.—A METHOD FOR EXTRACTING SQUARE ROOT

The following description assumes that the calculator keyboard has ten columns.

1. Set radicand on the keyboard with regard to the following rule: If the radicand has an even number of digits left from the decimal point, set the first digit in the extreme left (that is: #10) column; if it has an odd number of digits, set the first digit in the column next to the extreme left column (that is: in column #9).

2. Shift carriage to the extreme right position and multiply by 5. Clear keyboard and counter dial.

3. Set counter control in negative.

4. Set 5 in Col. 9 of keyboard; touch minus bar; set 1 in Col. 10; touch minus bar; set 2 in Col. 10; touch minus bar; set 3 in Col. 10; touch minus bar. Follow this procedure until ringing of the bell indicates an oversubtraction. Actually 05, 15, 25, 35, etc. have been subtracted from five times the radicand. When the oversubtraction occurs, touch the positive key to eliminate the oversubtraction, and shift the carriage one place to the left. Leave in the number in Col. 10 (extreme left column) of the keyboard. Remove number 5 from Col. 9 and set number 5 in Col. 8. Subtract again 05, 15, 25, 35, etc. (that is: set 0, 1, 2, 3, etc. in Col. 9) until an oversubtraction occurs. Repeat the procedure until the square root appears in the counter dial to the accuracy required.

Check: Radicand divided by square root equals square root.

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ELECTRIC ANALOG FOR TRIANGULATION ADJUSTMENT

By Hsuan-Loh Su[1]

SYNOPSIS

Various methods based on the principle of electric analogy for triangulation adjustment problems are suggested. It is shown in one of the examples that for a triangulation involving 8 angle equations and two side equations, the adjustment problem can be solved by one operator within a few days.

INTRODUCTION

The analogy between a level net and an electric network has been utilized to solve the level net adjustment problems.[2,3] The analogous electric network is generally known as the analog computer. A similar analogy will be established between a triangulation network and an electric network, which can be constructed for the solution of triangulation adjustment problems.

The principal advantages of an analog computer are simplicity, reliability, flexibility and, most important of all, economy. The methods used to achieve them, however, are not obvious. In this paper, a successful type of d.c. analog will be discussed.

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1 Imperial Coll., London Univ., London, S.W. 7.

2 "Electric Analog for Level Net Adjustment," by H. L. Su, Proc. ASCE, Nov., 1957, Paper 1443.

3 "Adjustment of a Level Net by Electrical Analogy," by H. L. Su, Empire Survey Review, vol. 13, no. 96, April, 1955.

Many other types of analog for adjustment problems can be deviced. For the level net, a structural analog has been suggested[4] and a hydraulic one was mentioned.[3] However, such mechanical analogs require the measurement of quantities in an imperfect model, and results from such measurement are not accurate enough.

Although there is no technical limit on the precision which can be attained by an electric analog, such method of computation will become an expensive proposition for a precision greater than four or five significant figures. The reason is that the necessary cost in increasing the precision beyond the mentioned range increases very rapidly. The analog certainly provides the most economic solution for a computation center, which wants a permanent set-up for frequent use. Nevertheless, if the computer is only to be used for one specific problem, it will be more economic to design an inexpensive analog and to combine it with a little calculation work to meet the requirements. The orthodox calculation methods are not of much use in such combination, and a method of current distribution will be suggested. Two other methods, which can also be applied here, were discussed elsewhere and will not be repeated here.[2]

PRINCIPLE OF ANALOGY

It is not difficult, although unnecessary in the present case, to establish a basic analogy between the electric network and the triangulation network as the resulting sets of linear equations are in both cases derived through minimizing a certain positive definite quadratic. Such simultaneous equations are generally known as Kirchoff's laws in the theory of electricity, and as the normal equations in an adjustment problem.

In the case of the adjustment of a level net, it has been shown that only ordinary resistors are involved in the analog. However, in a general "least square" problem, the normal equations may require some reactances differing 180° in phase in an a.c. network or some "negative resistors" in a d.c. network. It is the purpose of the present article to discuss such cases.

A set of normal equations in the form of matrices can be symbolically represented as follows:

$$A\,X = B \qquad (1)$$

in which A is the matrix of the coefficients, X the column matrix of the unknowns, and B the column matrix of the constants.

An electric network which is analogous to the triangulation network can be defined by Kirchoff's first law

$$G\,V = I \qquad (2)$$

in which G is the matrix of the conductance or admittance, V the column matrix of the unknown potentials, and I the column matrix of the feed-in currents.

It should be noted that the matrix G is symmetrical about the principal diagonal according to the theorem of reciprocality. If G and A, which is also symmetrical, as well as I and B are numerically equal or differ only by a constant multiplier, then the analogy between Eq. 1 and Eq. 2 is established.

4 "Relaxation Methods in Engineering Science," by R. V. Southwell, Oxford Univ. Press, 1940.

However, as I is fed from an outside source, the matrix G should have the property that the sum of the elements in any row, hence any column, must vanish.

Now consider the following equation

$$C\,X' = D \quad \text{.....................} \quad (3)$$

in which C denotes the matrix $\begin{bmatrix} A & E \\ E & \infty \end{bmatrix}$, which is one rank higher than A in Eq. 1; X' denotes the column matrix $\{X, x_0\}$; and D the column matrix $\{B, b\}$ with x_0 being zero and b an indefinite value. The vector E is added in order to make the sum of the elements in any row or column vanish. Eq. 3 will be referred to as the revised normal equation.

It can be seen that an analogy exists between Eq. 2 and Eq. 3 with C and D proportional to G and I respectively. The term x_0 in X' corresponds to V_0, the potential of the earth, and the infinity element in C conforms with the fact that the admittance or the conductance of the earth is unlimited. As a consequence, b being a current fed into the earth can assume any value.

Through the prior manipulation, the analogy between a triangulation network and an electric network is established. It is only necessary then to select a suitable multiplier for C and another for D to complete the electric transformation.

METHOD OF CURRENT DISTRIBUTION

Two methods of successive approximations have been dealt with elsewhere.[2] The present method is independent of these two and has the advantage that only currents are involved in the process of calculation. These three methods can be applied simultaneously to achieve rapid convergence.

From Ohm's law, which is valid for any branch in the circuit,

$$V = I\,Z + E \quad \text{..................} \quad (4)$$

in which V is the potential across the branch, I represents the current in the branch, Z denotes the impedance, and E is the applied e.m.f. As both Z and E are constant in the problem, the finite differentiation will give

$$\Delta V = Z\,\Delta I \quad \text{....................} \quad (5a)$$

or

$$\Delta I = Y\,\Delta V \quad \text{....................} \quad (5b)$$

in which Y is the reciprocal of Z and known as admittance.

According to Kirchoff's first law,

$$\sum \left(I + \Delta I\right) = \sum I + \Delta V \sum Y = 0 \quad \text{..............} \quad (6)$$

in which I is the assumed current and ΔI is the amount of current in each branch required to balance the flow-in and flow-out at a joint.

Hence at a joint, $\Delta V = -\sum I \,/\, \sum Y$ and $\Delta I == Y \Delta V = s\left(-\sum I\right)$ where $s = Y / \sum Y$ and will be called the distribution factor.

All V's will be assumed to be zero at the beginning of the calculation, and consequently, there should be no current flowing in the network at all. This assumption certainly satisfied Kirchoff's second law. Then V's are systematically increased or decreased until there is no excess current at any joint, and this can be achieved by distributing the unbalanced currents. It should be noted that in every step, Kirchoff's second law is always satisfied.

The criterion of a correct solution is that there exists no appreciable excess current at any joint in the network, for example, there is no need for further distribution. The interpretation of "appreciable" entirely depends on the precision required in a particular problem. For instance, if three significant figures are sufficient, and if the fourth figure already shows a sign of stability, this indicates that a solution with a precision either equal to or more than the requirement has been achieved. However, if a little discrepancy on the third figure is tolerable, then the "appreciable" level can be left there and there will be no need to go into the fourth or fifth figure.

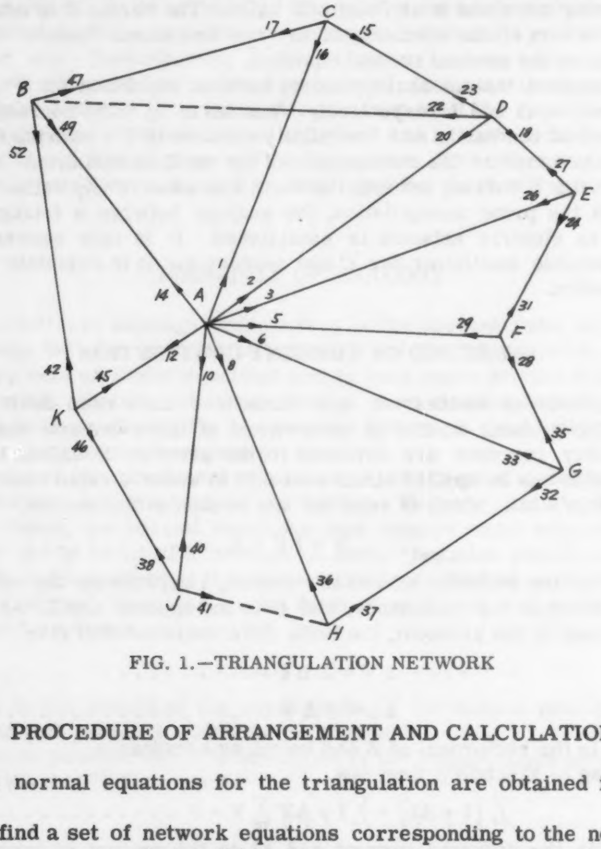

FIG. 1.—TRIANGULATION NETWORK

PROCEDURE OF ARRANGEMENT AND CALCULATION

(1) The normal equations for the triangulation are obtained in the usual way.

(2) To find a set of network equations corresponding to the normal equations, it is necessary that suitable dummy equation should be determined so as to make the sum of elements in any row or column in the square matrix vanish.

(3) A circuit diagram is drawn from the information given by the network equations, such as Eq. 2 or Eq. 3. The values of the conductance or the admittance of any branch of the circuit should be written down.

(4) The resistance or the reactance is then calculated. Suitable precision should be maintained in the reciprocation.

(5a) In the experimental method it is necessary to find an appropriate multiplier for all the resistors and another for the feed-in currents such that the measurement of the voltages of the network can be performed with accuracy and ease.

(5b) In the calculation method, the distribution factors should then be calculated. The initial currents are obtained from either the assumed or the experimentally determined voltages. The method of current distribution can then be applied and continued until the required precision is achieved.

(6) The results should finally be checked by calculation, which only involves simple arithmetic. If inconsistent voltages appear in the calculation method, redistribution will become necessary.

EXAMPLE 1

The first example chosen in an actual adjustment problem which, in spite of its simplicity, can fully illustrate the technique of the proposed method.

The triangulation network is shown in Fig. 1. It can be seen that the number of observed directions is 36, the number of the stations is 10, the number of full lines (with both directions observed) is 19, and the number of the half lines (with only one direction observed) is 2. Therefore, the number of angle conditions should be 19-2-10+1=8 and the number of side conditions is 19-20+3=2. The two side equations are obtained with station A and D as the pole respectively.

The normal equations are as follows

$$\begin{pmatrix} +6.00, & -2.00, & . & . & . & . & . & -2.00, & -4.29, & -23.49 \\ -2.00, & +6.00, & -2.00, & . & . & . & . & . & +5.40, & +3.88 \\ . & -2.00, & +6.00, & -2.00, & . & . & . & . & -7.10, & . \\ . & . & -2.00, & +6.00, & -2.00, & . & . & . & +11.22, & . \\ & & . & -2.00, & +6.00, & . & . & . & -4.67, & . \\ . & . & . & . & . & +6.00, & -2.00 & . & -6.27, & . \\ & & & & & -2.00, & +6.00, & -2.00, & +4.72, & +5.59 \\ -2.00, & . & . & . & . & . & -2.00, & +6.00, & -1.90, & -18.13 \\ -4.29, & +5.40, & -7.10, & +11.22, & -4.67, & -6.27, & +4.72, & -1.90 & +112.934, & +18.78 \\ -23.49, & +3.88, & . & . & . & . & +5.59, & -18.13, & +18.78, & +427.04 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \\ X_9 \\ X_{10} \end{pmatrix} = \begin{pmatrix} +18.50 \\ +9.20 \\ -12.20 \\ +20.70 \\ -21.50 \\ +16.20 \\ -0.10 \\ -14.20 \\ +17.90 \\ -54.30 \end{pmatrix} \quad \cdots\cdots \quad (7)$$

The revised normal equation will therefore be as follows

$$\begin{pmatrix} & & & & & & & & & & +25.76 \\ & & & & & & & & & & -11.28 \\ & & & & & & & & & & +5.10 \\ & & & & A & & & & & & -13.22 \\ & & & & & & & & & & +0.67 \\ & & & & & & & & & & +2.27 \\ & & & & & & & & & & -12.31 \\ & & & & & & & & & & +18.03 \\ & & & & & & & & & & -128.82 \\ & & & & & & & & & & -413.69 \\ +25.76, & -11.28, & +5.10, & -13.22, & +0.67, & +2.27, & -12.31, & +18.03, & -128.82, & -413.69, & \infty, \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \\ X_7 \\ X_8 \\ X_9 \\ X_{10} \\ X_0 \end{pmatrix} = \begin{pmatrix} \\ \\ \\ B \\ \\ \\ \\ \\ \\ \\ b \end{pmatrix} \quad \cdots\cdots \quad (8)$$

In Fig. 2, the actual analog network is shown. The ordinary positive resistors are represented by full lines and the negative ones by dotted lines. The feed-in currents are also schematically depicted on Fig. 2. It should be noted that the actual values of all resistors have been multiplied by 2,000 so that the low-rating laboratory resistors can be employed. The feed-in currents, on the other hand, are divided by 10 for a similar reason. The experimental results are collected and shown subsequently in Fig. 6.

Although the current distribution method is essentially an ancillary to the analog, it is an independent means. In order that this method can be clearly understood, the following calculation will be started from the very beginning, that is, without any assistance from the analog.

At joint 1, the unbalanced current is equal to the feed-in of -18.5 units (in fact, -1.85 ma). At this stage, there is no need to involve three figures in the calculation and the unbalanced amount is taken as simply -18, which requires balancing currents totalling +18 units. (Currents flowing away from a joint is considered positive at that joint). This total is to be provided by the various lines forming the joint in the proportion of their distribution factors, which are shown in Fig. 4. Therefore,

$$
\begin{array}{llrcr}
\text{on} & 1\text{-}2\,, & 18/3 & = & 6 \\
 & 1\text{-}8\,, & & & 6 \\
 & 1\text{-}9\,, & 0.714\times 18 & = & 13 \\
 & 1\text{-}10, & 3.9\times 18 & = & 70 \\
 & 1\text{-}E\,, & -4.29\times 18 & = & -77 \\
\hline
 & & & \sum I = & +18
\end{array}
$$

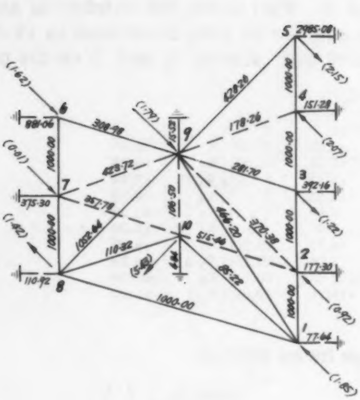

FIG. 2.—ANALOG NETWORK

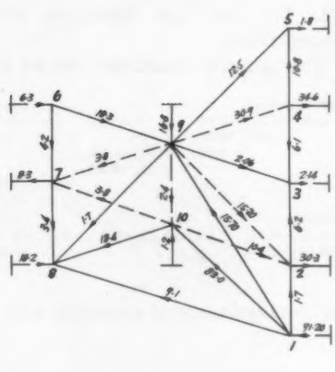

FIG. 3.—EXPERIMENTAL RESULTS

At joint 2, in addition to the feed-in of -9.20 units, there is also a current of -6 units flowing towards joint 2 from joint 1. Therefore, the balancing currents should amount to -(-6-9.2)= say +15. Hence, on

$$
\begin{array}{llrcr}
2\text{-}1\,, & -6 + 15/3 & = & -1 & \\
2\text{-}3\,, & 15/3 & = & 5 & \\
2\text{-}9\,, & -0.90\times 15 & = & -14 & \\
2\text{-}10, & -0.65\times 15 & = & -10 & \\
2\text{-}E\,, & 1.89\times 15 & & +28 & \sum I \text{ after distribution} = +8
\end{array}
$$

The procedure is repeated at other joints and the calculation is continued until the out of balance currents become negligible at any particular stage. Such calculation is best performed in association of a sketch as Fig. 4 with results written down on a sketch as Fig. 5.

It can be seen that after the first round of distribution, the results already show a tendency to converge to the correct results. When the analog results such as those shown in Fig. 3 are used as the first set of approximations, it will be found that the unbalanced currents are already correct up to three or four figures at least, and only a few rounds of distribution can improve the results up to the precision of more than five significant figures.

The method of checking and the subsequent re-distribution, though seldom necessary due to the reliability of the analog, is very simple in principle and in operation. It should be noted that the method of current distribution is a method of successive approximations, which can be started on any set of assumed initial values of potentials or currents. In the illustration, the initial values of the potentials are assumed to be zero. If some mistakes are found, it is only necessary to start the distribution process from the wrong results again, and there is no need to go through the previous calculation. Therefore, the calculation method can serve two purposes, (1) to obtain more significant figures, which will be too expensive for an analog; and (2) to check the experimental as well as the arithmetic errors in the results.

Many other artifices on the numerical calculation can also be employed to save time and to avoid mistakes, but will not be dealt with here because by doing so, this paper will be considerably lengthened.

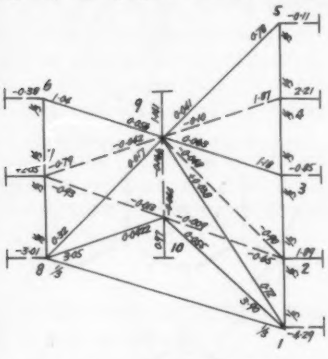

FIG. 4.—DISTRIBUTION FACTORS

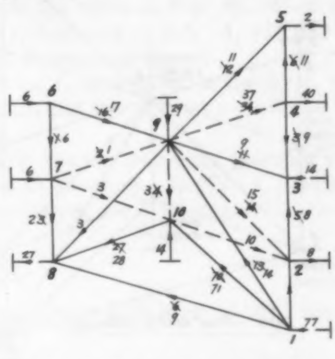

FIG. 5.—CURRENT DISTRIBUTION

In Fig. 6, the experimental results and the calculation ones are compared. The voltages at the ten joints were measured with a precision potentiometer, and the obtained results only reflect the tolerance of the resistors, which was around 0.1% to 0.01%.

EXAMPLE 2

In distinction to Example 1, which is a general adjustment problem, this second example is a problem involving angle equations only. Fig. 7 shows a typical portion of such triangulation network. For the sake of simplicity in explanation, the weights of all observed directions are assumed to be equal. A problem with unequal weights can be similarly treated.

The normal equation corresponding to the i-triangle is therefore

$$6k_i - 2k_1 - 2k_2 - 2k_3 + w_i = 0 \quad \ldots\ldots\ldots\ldots\ldots \quad (9)$$

in which k's are the correlates, and k_i the one corresponding to w_i the i-th angle excess, and hence also to the i-th triangle. Since the sum of the coefficients of the unknowns vanishes, there is no need to introduce a dummy for Eq. 9. The corresponding circuit equation can be immediately written as follows

$$\left(\sum \frac{1}{R}\right) V_i - \frac{V_1}{R_1} - \frac{V_2}{R_2} - \frac{V_3}{R_3} = I_i \quad \ldots\ldots\ldots\ldots\ldots \quad (10)$$

in which $R_1 = R_2 = R_3 = 1/2$ units, therefore, $\sum \frac{1}{R} = 1/6$. (For a problem of unequal weights, these resistances will also be unequal.)

Eq. 10 shows that for problems involving angle equations only the analog network will be a d.c. circuit involving ordinary resistors, with equal magnitudes for problems of equal weights. It can be seen that if the i-th triangle

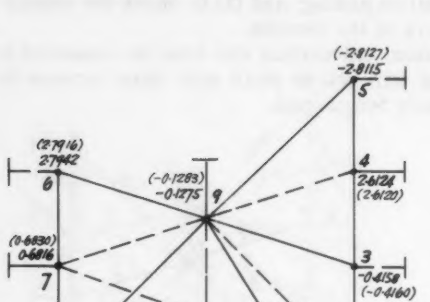

FIG. 6.—COMPARISON OF RESULTS

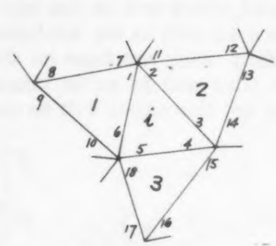

FIG. 7.—TYPICAL NETWORK

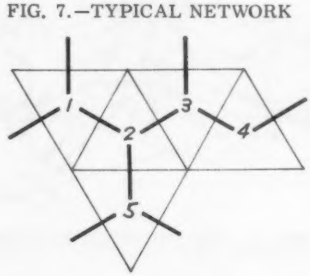

FIG. 8.—ELECTRIC TRANSFORM

happens to be on the boundary of the triangulation net, for example, there is no triangle 3, then k_3 will vanish. This simply means that V_3 should be kept zero and consequently this joint should be earthed. Some interesting conjugate properties between the triangulation and the analog can also be noted. (1) An internal triangle in the triangulation corresponds to an internal joint in the analog. By internal joint we mean a joint without an earthed branch. (2) An external triangle in the triangulation network corresponds to an external joint in the circuit. A triangle with two external sides corresponds to a joint with two earthed lines, although in the actual analog such lines are combined into one. (3) The sides forming the i-th triangle in the triangulation corresponds to the connecting lines forming the i-th joint in the analog.

These characteristics of a problem involving angle equations are only very useful in the experimental and the calculation methods. Since all resistors are equal in magnitude for a problem of equal weights, very high accuracy of the resistance value can be conveniently attained. Normally, it is sufficient to delegate such a problem completely to the analog from which five figures results can be obtained. For the case of unequal weights, a few resistors will have different magnitudes and can be wound with resistance wires.

In the calculation method, the distribution factors are always positive, as only ordinary resistances are involved, and will be equal to one third always in the case of equal weights.

The triangulation network for illustration is schematically shown in thin lines on Fig. 8, and the thick lines represent the resistors connecting the five internal joints and the earth. The values of the angle excess for the triangle 1, 2, 3, 4, and 5 are +12, +10, -8, +28 and -14 respectively.

In an actual analog, the convenient value for the resistors will be 100 ohms, and the feed-in currents can be +1.2, +1.0, -0.8, +2.8 and -1.4 ma. Consequently, the measured voltages (in mv) should be multiplied by a factor of 100/20 = 5 so as to make them numerically equal to the values of the correlates.

Another method of arrangement of the analog is shown in Fig. 10, where the feed-in currents are substituted by e.m.f.'s imposed on the network. The magnitudes of these emf's are marked beside the cells in Fig. 10. When the magnitude of the resistors is taken to be unity, currents of 6.5, -4, +14 and -7 units are thus produced at joint 1, 2, 3, 4, and 5 respectively.

In order to show that the second arrangement is equivalent to the first, the following calculation will be started by distributing the currents produced by those emf's.

The initial currents produced by the cells are the same as the emg's in magnitude because all resistances are taken to be unity. At joint 4, there is an unbalanced current of +14 units. To balance this, an equal amount of flow-in must be supplied

$$\Delta I = -(+14)/3 = -5$$

Therefore, there must be a flow-in of 5 units from each branch. The situation is now as follows

on	4X ,	0 - 5	= -5
	4-Y,	+14-5	= +9
	4-3,		-5

After the distribution, there is still an unbalanced current of -1 unit, which is tolerable at this stage.

At joint 5, $\Delta I = -(-7 +5 -5)/3 = +2$. After distribution,

on	5-X,	-7 +2	= -5
	5-Y,	+5 +2	= +7
	5-2,	-5 +2	= -3
	Total		= -1

At joint 1, $\Delta I = -(+6)/3 = -2$. Therefore,

1-X,	0-2	= -2
1-Y,	+6-2	= +4
1-2,	0-2	= -2
Total		= 0 after distribution.

This process of current distribution can be continued until sufficient precision has been attained. The final results in three significant figures are shown in Fig. 11. Fig. 10 shows the actual analog circuit, and the above calculation is recorded in Fig. 9.

EXAMPLE 3

In view of the simplicity of the analog in Example 2, it is worth while to investigate whether the method employed can be generalized and extended to all kinds of problems. This is the aim of the third example.

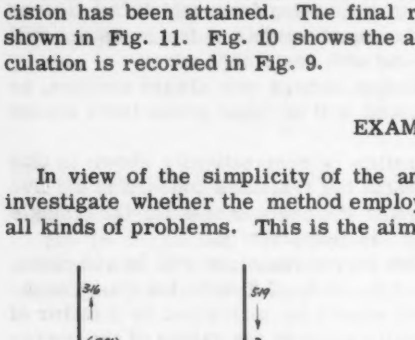

FIG. 9.—CURRENT DISTRIBUTION

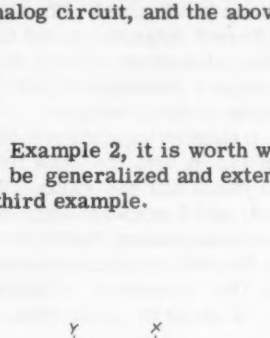

FIG. 10.—ANALOG CIRCUIT

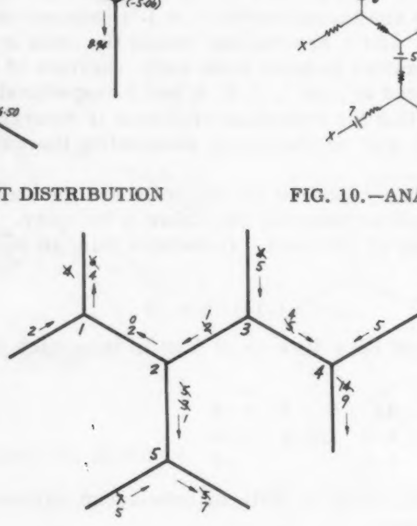

FIG. 11.—FINAL RESULTS

As previously mentioned, an accurate analog for general use may be quite expensive and it is often desirable to look into the possibility of reducing the capital as well as the operation cost. Fortunately, there are normally much more angle equations than side equations in a triangulation adjustment. It has been shown in Example 2 that angle equations are very simple and a high precision is not difficult to be effected with an analog. This suggests a method of treating the adjustment problem.

It should be noted that the analog constructed for Example 1 can easily be adapted to any problem involving a set of linear equations, whether symmetrical or not, and no advantage has been taken of the properties characterizing a triangulation problem. Naturally, it will be expected that when these characteristics are taken into account, some simplification can be achieved.

Suppose x_9 and x_{10} vanish from Eq. 7. Then the general problem degenerates into a problem involving the angle equations only, and therefore can be solved by a simple analog as described in Example 2.

Obviously, the first solution of the simplified problem cannot satisfy the normal equation of the general problem. Now, from the last two rows of Eq. 7, the values of x_9 and x_{10} are substituted into other eight rows, the elements of the column matrix on the right hand side will be altered. This means that the currents fed into the analog for the simplified problem should be adjusted to their new values. Such change will produce another set of solutions to the simplified problem, which will be referred to as the second solution.

Consequently, there will be second values of x_9 and x_{10} corresponding to the second solution of x_1, $x_2 \ldots$, x_8. When they are substituted into the first eight rows of the normal equation, a new column matrix will appear and produce the third solution. By repeating such operation, it is possible to obtain a final solution which satisfies the normal equation of the general problem.

The aforementioned process of iteration can be carried out by calculation alone, that is, by the current distribution method or the other two discussed in the previous article; or by two simple analogs; or by combination of one simple analog with a calculation method. There is nothing novel in the first method. In the second, the analog for the general problem will be split into two, one for solving the angle equations and the other for solving the side equations. The technique developed previously for setting up a general analog can be applied to the arrangement of an analog for side equations only. If Example 1 is considered again, the side equations can be written as follow:

$$\begin{pmatrix} 112.93, & 18.78 \\ 18.78, & 427.04 \end{pmatrix} \begin{pmatrix} x_9 \\ x_{10} \end{pmatrix} = \begin{pmatrix} 17.90 + F_9 \\ -54.30 + F_{10} \end{pmatrix} \quad \ldots\ldots\ldots \quad (11)$$

where F_9 and F_{10} denote the influence of the change of $x_1, \ldots . x_8$ upon x_9 and x_{10} respectively. The required network equation will then be

$$\begin{pmatrix} 112.93, & 18.78, & -131.71 \\ 18.78, & 427.04, & -445.82 \\ -131.71, & -445.82, & \infty \end{pmatrix} \begin{pmatrix} x_9 \\ x_{10} \\ x_0 \end{pmatrix} = \begin{pmatrix} 17.90 + F_9 \\ -54.30 + F_{10} \\ \text{Indeterminate} \end{pmatrix} \quad \ldots\ldots\ldots \quad (12)$$

Eq. 12 requires one negative resistor of 18.78 units between joint 9 and joint 10, both of which are earthed through a positive resistor of 131.71 and 445.82 units respectively.

The network for the angle equations is very simple. Joint 5 and joint 6 will become the ends of a chain consisting of joint 4,3,2,1,8,7, (in this order) each of which is earthed through a positive resistor of a standard value, say 100 ohms. The end joints will be earthed either through two resistors of 100 ohms or through one of 50 ohms.

Since the network defined by Eq. 12 still looks a little complicated, sometimes it will become preferrable to discard it and to use any calculation method instead, particularly when the number of the side equations is small.

In order to show the power of this method, the results from the analog for angle equations only have been tabulated in Table 1 in comparison with the exact value. It should be mentioned that these values are obtained by feeding in currents accurate up to the third figure, therefore only three figures are shown in Table 1 although the potentiometer gives answers in five figures. (The results in five figures are the "exact" values.) Furthermore, there is no need to expend any effort on the fourth and the fifth figures on the first trial.

The corresponding values of x_9 and x_{10} are -0.179 and -0.071 compared with the final values of -0.1275 and -0.0032 respectively. The currents to be fed into joint 1,2, . . , 8 for a second solution should consequently, be changed to +16.07, +10.34, +13.47, +22.70, -22.34, +15.08, +1.04, -15.83 units respectively. The measurement of the second set of voltages can then begin.

From the results shown in Table 1, it can be seen that they are already quite near the final answer. In fact, after three trials, adding one more significant figure each time, the results are already correct up to the fourth figure. The fifth figures are then obtained with method of current distribution to check the results.

This method is particularly useful in this problem because there are only two side equations. When the number is greater than two, it will be more rewarding to have another analog to solve the side equations.

TABLE 1.—RESULTS OF FIRST TRIAL ON SIMPLIFIED ANALOG COMPARED WITH THE EXACT VALUES

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
+3.68	+2.68	-0.36	+2.64	-271	+2.92	+0.71	-0.92
+3.5407	+2.6912	-0.4158	+2.6124	-28115	+2.7942	+0.6816	-1.0093

ECONOMIC CONSIDERATION

As mentioned before, there is no technical limit of precision of the analog so far as the triangulation problems are concerned. However, when the question of economy is taken into account, there will be an optimum limit. The writer's experience shows that the precision of $\pm$ 0.1% can be achieved with ease, and that of 0.01% will require special care in either winding the resistors or checking those supplied by a manufacturer on a special order. If only 1% is sufficient as in the case of the combination method, then such analog can be set up with very simple equipments and not much effort.

Since the electric meters reading five figures are commercially available, the accuracy of an analog depends to a very great extent upon the accuracy of the resistors and the precision of their calibration.

The time required to set up an analog for the general problem, that is, for solving all equations simultaneously, is around two days, whereas that required for the analog to solve angle equations only is a matter of a few hours. The time for adjusting the feed-in current and for measurement of voltages is comparatively negligible. In fact, all the experiments, including the accompanying calculation mentioned in this paper, were finished within a week by one operator.

It can be observed now that the analogs discussed in this paper do not involve many difficult electric problems, that a civil engineer is unable to deal with. Yet the cost of such analogs is extremely low in comparison with any of the other items of expense involved in a triangulation project.

CONCLUSIONS

The suggested methods can be grouped into two categories: the calculation method and the experimental method. The calculation method can be subdivided into three categories: the current distribution method suggested in the present paper, the current adjustment method and the potential adjustment method, two of which were dealt with elsewhere.[2] Since all of them are methods of successive approximation, it is very easy to transform them into methods of successive corrections, or to combine both into a method of alternative approximations and corrections. Furthermore, there are many artifices in the numerical manipulation which will speed up the process of convergence, such as over-estimation, group-distribution, and group-adjustment. These refinements cannot be presented within the text of the present article.

The experimental method also has three subdivisions: the general analog, for solving any set of simultaneous equations, a simplified analog for angle equations only in combination with another for side equations, and a simplified analog which is alone.

It should be noted that all these methods, experimental or numerical, are mutually complementary in accordance with the prevailing economic consideration. It is difficult to set a general rule of choice, and the selection of such methods will to a large degree depend upon not only economic and technical consideration but also personal preference. It is often found that the method with which an engineer is most conversant is the best method for himself. However, one thing is quite certain. The electric analog certainly provides an economic and effective means of solving triangulation problems with or without the assistance of numerical calculation.

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Journal of the
SURVEYING AND MAPPING DIVISION
Proceedings of the American Society of Civil Engineers

AIRBORNE SURVEYING WITH DOPPLER SYSTEMS[a]

By L. R. Chapman, Jr.[1]

SYNOPSIS

Self-contained navigation systems based on the familiar "Doppler effect" provide precise, automatic direction of an aircraft over a predetermined flight path, with accurate side-lap and over-lap control. The advantages resulting from fewer reflights, and elimination of ground-based aids, are reflected in better accuracies and lower costs of Doppler survey operations.

Since 1948, when the first airborne Doppler navigation system was successfully operated in flight, the inherent accuracy and operational efficiency of these self-contained navigation devices have led to their use in a variety of applications. Other than the basic use as prime navigation equipment, some of the more important uses have included those in jet-stream operations, in basic meteorological research, in synoptic weather observation, and in radar stabilization. One of the fields in which the Doppler equipment has been recently used with success is that of airborne reconnaissance. Preliminary results have indicated that airborne geo-magnetic and photo-mapping operations can greatly benefit from the use of this self-contained navigation equipment.

Aviators have always been concerned with the problems of how to measure the ground speed and drift angle of an aircraft without aid from the ground. Optical drift sights, bomb sights, radar, and the Bellamy drift technique have been

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[a] Presented at the October, 1958, ASCE Convention in New York, N. Y.

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used in the past. All of these methods have serious operational limitations. With the advent of Doppler self-contained navigation systems, pilots have available, continuously, highly accurate measurements of the ground speed and drift angle of the vehicle.

The technique which makes possible the sensing and measurement of ground speed and drift angle utilizes what is commonly referred to as the "Doppler effect". The Doppler phenomenon itself has, of course, been well known for many years. As a very elementary example of the Doppler principle, let us suppose that we have a stationary transmitter-receiver. The transmitter is emitting a signal toward a moving "target." The signal reflected back to the stationary transmitter-receiver, is at a different frequency from that of the transmitted signal, and the frequency difference is known as the Doppler shift. This frequency shift is proportional to the relative velocity between the transmitter-receiver and the target.

In airborne Doppler systems, the transmitter-receiver is located in an aircraft flying over the ground. The microwave energy is emitted downward from a moving transmitter and is reflected by a stationary "target," the earth's surface beneath the aircraft. A signal beamed forward and down undergoes a Doppler shift and is at a higher frequency, as detected by a receiver on the ground. The energy is reflected at this new (higher) frequency by the earth, which may be considered a new transmitter. The receiver on the aircraft is, of course, moving relative to the earth, and the signal received at the aircraft undergoes another Doppler shift, upward in frequency. The Doppler frequency is still proportional to the velocity of the aircraft, and can be translated to give an indication calibrated directly in knots.

Multiple beams are used to determine the drift angle of the aircraft. One beam of energy is confined to an area to the left of the fore-aft axis of the aircraft, and another beam of energy is confined to an area to the right. Where cross winds are present, an aircraft's fore-aft axis does not coincide with the line of flight. The beam, which is closer to the ground track, will undergo a greater frequency shift. In order for both beams to turn to the same frequency shift, it is necessary to rotate the antenna system in azimuth so that it is always alined with the path of flight, and so that the bisector of the angle formed by the two beams of energy coincides with the direction of the velocity vector of the aircraft.

In brief, by measurement of relative frequencies, the ground speed of an aircraft can be determined. By rotating a multiple-beam antenna until equal Doppler frequencies are obtained from beams displaced symmetrically on each side, drift angle can be measured.

During the development of the Doppler navigation systems it was necessary to accurately measure, by independent means, the true velocity and drift of an aircraft. Photogrammetry techniques were and are used for this task. Accuracies on the order of 0.1% for velocity and 0.1° for drift are required and are routinely obtained, using specially surveyed test ranges. It occurred that this procedure could be reversed, and that the natural capabilities of the Doppler navigation equipment could be used for highly precise aerial mapping and related missions. The systems available today operate anywhere in the world, over land, water, desert or mountains, at all altitudes, and are completely independent of ground-based aids. These self-contained equipments provide accurate flight-line control, and supply outputs of ground speed and drift angle for camera stabilization, intervalometer control, and image motion compensation.

A Doppler system, as required for aerial mapping, consists of a Doppler radar, a navigational computer, and a heading reference. The Doppler radar determines actual ground speed and drift angle. The output of the heading reference is added to drift angle to obtain the ground track angle (relative to north), or direction of travel to the aircraft. It is the function of the computer to integrate ground speed to obtain distance travelled, and to derive steering signals, which will maintain the aircraft on the desired flight path. In a typical surveying flight pattern, the aircraft can be automatically controlled to maintain close control of length of lines, and of side-lap. The typical accuracies of the Doppler output are 0.2% for velocity and 0.2% for drift angle. The heading reference, with limiting accuracies of, say, 0.25°, therefore, introduces the largest errors. Fortunately, the typical surveying pattern does not require accurate orientation of the successive flight lines; it is only necessary that the lines be parallel. Therefore, a carefully compensated magnetic heading reference with high quality gyroscopic stabilization can be used with good results. Changes in magnetic variation over the surveyed area will result in slightly curved flight lines, which will still be parallel. Depending on the lengths of the flight lines, accuracy can be improved somewhat by operation of the compass as a free gyro, after initial magnetic or optical alinement. Special gyros having various drift rates of 0.25° per hr, can be obtained for this purpose. In order to utilize fully the capability of the Doppler radar, however, a more accurate heading device should be used. Sun tracking devices offer the best immediate solution to this problem. With present off-the-shelf components, accuracies of ± 2,000 ft in side-lap control can be obtained for 50-mile legs. With carefully calibrated equipment somewhat better accuracy is possible.

Truly precise control of picture spacing is inherent in this system, due to the accurate measure of velocity. With present radar altimeters, and a small V/H computer, control of overlap percentage to ±0.4% can be obtained. This technique provides absolute control of scale and results in greatly simplified photo rectification processes. Additional gains result from photos which are arranged in true checkerboard fashion.

The extreme precision of a Doppler radar as a distance-measuring device suggests its use as a surveyor's chain, particularly in accurate location of control points in uncontrolled areas. A Doppler equipped aircraft can be used to "chain" the distances between a desired control point and two known points, so that the unknown position can be determined by triangulation. By using a carefully calibrated Doppler system, and by making several repeated flights to add statistical advantage, a point can be located within an accuracy of 100 ft after 100 mile triangulation legs.

By combination of Doppler and inertial sensors in a closed-loop fashion, the advantages of both can be exploited for a very precise system. As well as providing a more accurate heading reference, this system appreciably reduces ground data-processing time. The accuracies of ground speed and drift angle outputs, which in Doppler systems are optimum only when averaged over a number of miles of travel, are provided on an instantaneous basis. Further, aircraft pitch and roll outputs are available with the accuracy to which photographs are normally rectified. Thus, it becomes feasible to stabilize camera mounts completely in roll, pitch, and azimuth with the desired precision.

A Doppler-equipped aircraft is engaged (as of October, 1958) in a commercial geo-magnetic surveying operation in the Sahara Desert, where control is practically non-existent. The advantages of the self-contained navigational aid are resulting in increased precision, in addition to the very sizeable economic savings over conventional surveying methods.

CONCLUSIONS

A new tool is available to the airborne surveying profession. The self-contained Doppler systems that have been described, lend themselves admirably to many phases of photo and geo-magnetic reconnaissance. The aircraft can be automatically controlled to provide precise, pre-determined flight paths, with uniform coverage. Control of side-lap and over-lap is far better than possible with conventional techniques. With triangulation measurements utilizing the accurate distance-measuring capability, control points can be located with remarkable accuracy. Camera stabilization in azimuth, roll, and pitch is, for the first time, possible with great precision. The advantages resulting from few reflights, simpler data analysis, reduced over-lap requirements, and no need for ground-based aids are reflected in the better accuracies and lower costs of Doppler survey operations.

These systems are available today. They can be installed in any kind of aircraft, and are usable anywhere in the world. It is certain that as their use becomes widespread, new applications and techniques will be developed, which will change the concepts of surveying as known today. The benefits which have been achieved in airborne navigation, in air-traffic control, and in meteorological research, will be extended to the fields of surveying and mapping.

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INTERSECTION OF STRAIGHT LINE WITH SPIRAL

By T. F. Hickerson,[1] F. ASCE

SYNOPSIS

In solving bridges or other structures along spiral curves connecting tangents with circles, one is confronted with the problem of locating precisely the points of intersection of straight lines (those through piers, say) with the center-line base spiral and the outer and inner parallel spirals as well.

By using the formulas with high-speed electronic digital computers, these intersections may be determined to a high degree of accuracy.

Let x and y be the coordinates of any point on the center-line spiral (Fig. 1) referred to the TS as origin. In Fig. 1, R is the variable radius of the center-line spiral (the base spiral) referred to any point distant l from the TS (at point A) and θ denotes the angle subtended by spiral arc AP.

At point C (the SC) arc AP = l_s and θ becomes θ_s (the so-called "spiral angle") and l_s is the total length of spiral connecting a tangent with a circular curve of degree D_c. Let W equal the radial distance from the base spiral to the outer (or inner) parallel spiral.

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The following expressions are well known:

$$l_s = 200\ (\theta_s/D_c) \quad \text{. . . . .} \quad (1a)$$

and

$$\theta = (l/l_s)^2\ \theta_s \quad \text{. . . . .} \quad (1b)$$

For an assigned θ, the actual values of the coordinates of a point on the base spiral are $l\,x$ and $l\,y$; where x and y (for the unit spiral)are given in Table 12, "Route Surveys and Design," McGraw-Hill Book Co., or by the expanded series:

$$x = \left(1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \frac{\theta^6}{9360}\right) \quad \text{. . . . .} \quad (2a)$$

and

$$y = \left(\frac{\theta}{3} - \frac{\theta^3}{12} + \frac{\theta^5}{1320}\right) \quad \text{. . . . .} \quad (2b)$$

Let the equation of the given line be designated by

$$y - y'' = m(x - x'') \quad \text{. . . . .} \quad (3)$$

where the origin of coordinates is at point A (the TS). Assume the line to

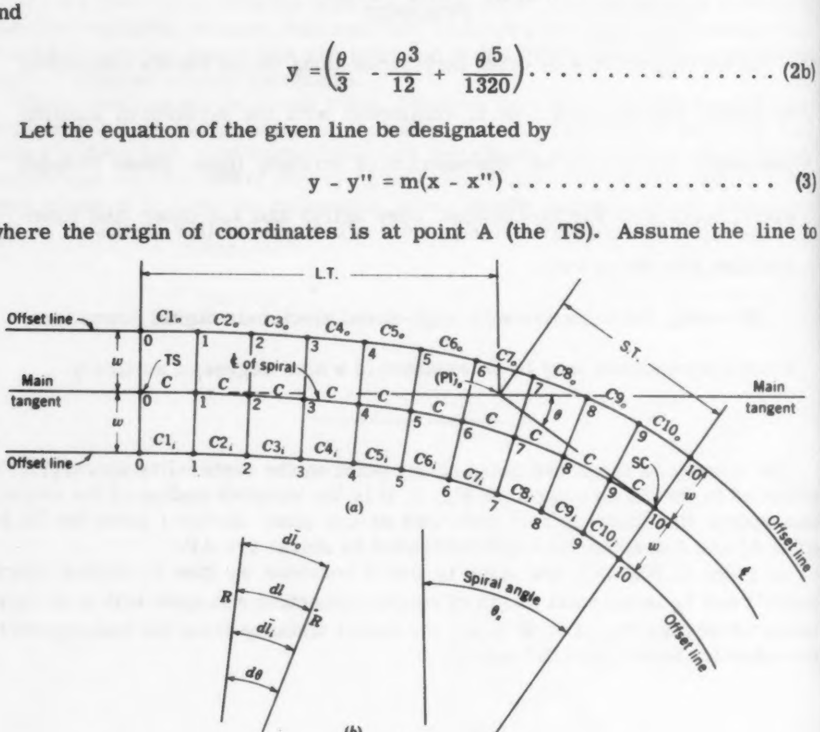

FIG. 1

cross the X-axis at a given point distant a from the origin, that is, at the point where $x'' = a$ and $y'' = 0$). Then Eq. 3 becomes

$$y = m\ (x - a) \quad \text{. . . . .} \quad (4)$$

At point 1, the intersection of the line with the base spiral, let the corresponding values of l and θ equal l_1 and θ_1, while the coordinates are $l_1\,x_1$ and $l_1\,y_1$ respectively.

Since both the TS and SC of the outer and inner spirals must be radially opposite the base spiral at these points (when normally they are not), their true spiral characteristics are slightly modified, thus causing a slight (but inappreciable) compounding at the SC.

From points 2' and 3' where the line crosses the outer and inner spirals, let radial lines of length W extend to points 2 and 3 of the base spiral. Hence (Fig. 1),

$$y_2' = W \cos \theta_2 - l_2 y_2 \quad \ldots\ldots\ldots\ldots \text{(5a)}$$

and

$$x_2' = l_2 x_2 + W \sin \theta_2 \quad \ldots\ldots\ldots\ldots \text{(5b)}$$

where l_2, θ_2, y_2, and x_2 apply to point 2 on the base spiral. Likewise, for the inner spiral, we have

$$y_3' = -(W \cos \theta_3 + l_3 y_3) \quad \ldots\ldots\ldots\ldots \text{(6a)}$$

and

$$x_3' = l_3 x_3 - W \sin \theta_3 \quad \ldots\ldots\ldots\ldots \text{(6b)}$$

Given the following data: l_s, θ_s, W, a, m(= slope of the line), and α (where $\tan \alpha = m$); the coordinates of the intersection points 1, 2' and 3' may be derived from the subsequent equations which are adapted to electronic computation.

Since the given line passes through points 1, 2', and 3', its equation is satisfied by the coordinates of these points; that is:

$$-(l_1 y_1 + m l_1 x_1) + m a = 0 \quad \ldots\ldots\ldots\ldots \text{(7)}$$

$$y_2' - m x_2' + m a = 0 \quad \ldots\ldots\ldots\ldots \text{(8)}$$

or

$$W \cos \theta_2 - l_2 y_2 - m(l_2 x_2 + W \sin \theta_2) + m a = 0 \quad \ldots\ldots \text{(9)}$$

$$y_3' - m x_3' + m a = 0 \quad \ldots\ldots\ldots\ldots \text{(10)}$$

or

$$-(W \cos \theta_3' + l_3 y_3) - m (l_3 x_3 - W \sin \theta_3') + m a = 0. \ldots\ldots \text{(11)}$$

Omitting subscripts for the moment, and reducing, Eqs. 7, 9, and 11 become

$$l(y + m x) - m a = 0 \quad \ldots\ldots\ldots\ldots \text{(12)}$$

$$l (y + m x) - [W(\cos \theta - m \sin \theta) + m a] = 0 \quad \ldots\ldots \text{(13)}$$

and

$$\mathrm{l}\,(y + m\,x) \;-\; [\mathrm{W}(\,m \sin\theta - \cos\theta) + m\,a] = 0 \quad \ldots\ldots\ldots \quad (14)$$

Substituting the expanded series for y and x into Eq. 13, we have

$$\mathrm{l}\left[\left(\frac{\theta}{3} - \frac{\theta^3}{12} + \frac{\theta^5}{1320}\right) + m\left(1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \frac{\theta^6}{9360}\right)\right]$$

$$- [\mathrm{W}(\cos\theta - m\sin\theta) + m\,a] = 0 \quad \ldots\ldots\ldots \quad (15)$$

For an approximation, Taylor's Theorem (given in treatises on calculus) will be used. Applying this theorem to Eq. 15, let $\theta_2 = \theta + \mathrm{K}$, where θ is the solution, that is, $f(\theta) = 0$, and θ_2 is an approximation. Then

$$\mathrm{K}\, f'(\theta) = f(\theta_2) = \mathrm{E} \quad \ldots\ldots\ldots \quad (16)$$

where E is the error.

Noting that $\mathrm{l} = (\mathrm{l_s}/\sqrt{\theta_s})\,(\sqrt{\theta})$, differentiating Eq. 15, neglecting terms θ^5 and over, and reducing, we get

$$f'(\theta) = (\mathrm{l_s}/\sqrt{\theta_s})\left[1/2\,\theta^{1/2} - \frac{1}{12}\,\theta^{5/2} + m\left(1/2\,\theta^{-1/2} - 1/4\,\theta^{3/2} + \frac{1}{48}\,\theta^{7/2}\right)\right]$$

$$+\left[\mathrm{W}(\sin\theta + m\cos\theta)\right] = \mathrm{E} \quad \ldots\ldots\ldots \quad (17)$$

For this approximation, put $\theta = \theta_2$ and divide both sides of Eq. 16 by $\sqrt{\theta}$ or $\sqrt{\theta_2}$. Then

$$\mathrm{K}\left\{(\mathrm{l_s}/\sqrt{\theta_s})\left[1/2 - \frac{1}{12}\,\theta_2^2 + m\left(1/2 - 1/4\,\theta_2^2 + \frac{1}{48}\,\theta_2^2\right)\right] + (\mathrm{W}/\sqrt{\theta_2})(\sin\theta_2\right.$$

$$\left. + m\cos\theta_2\right\} = (\mathrm{l_s}/\sqrt{\theta_s})\left[1/3\,\theta_2 - \frac{1}{12}\,\theta_2^3 + \frac{1}{1320}\,\theta_2^5 + m\left(1 - \frac{1}{10}\,\theta_2^2 + \frac{1}{216}\,\theta_2^4\right.\right.$$

$$\left.\left. - \frac{1}{9360}\,\theta_2^6\right)\right] - (1/\sqrt{\theta_2})\,[\mathrm{W}(\cos\theta_2 - m\sin\theta_2) + m\,a] \quad \ldots\ldots \quad (18)$$

Eq. 18 can be solved directly and the correction K found. The right-hand side of Eq. 18 has to be taken to more terms since greater accuracy is required, for this must eventually be computed to almost zero, whereas the coefficient of K in the left-hand side remains comparatively large, and as K approaches zero, such accuracy is not required. Then $\theta = (\theta_2 - \mathrm{K})$ = recurring value of θ_2.

For the sake of brevity, multiply by $\sqrt{\theta_s}$ and let

$$\mathrm{C_1} = \mathrm{l_s}\left[1/2 - \frac{1}{12}\,\theta_1 + \frac{m}{\theta_1}\left(1/2 - 1/4\,\theta_1^2 + \frac{1}{48}\,\theta_1^4\right)\right]$$

$$\mathrm{C_2} = \mathrm{l_s}\;\; 1/2 - \frac{1}{12}\,\theta_2 + \frac{m}{\theta_2}\left(1/2 - 1/4\,\theta_2^2 + \frac{1}{48}\,\theta_2^4\right)\Bigg] \quad \ldots \quad (19)$$

$$+ \mathrm{W}(\sqrt{\theta}_s/\sqrt{\theta_2})(\sin\theta_2 + m\cos\theta_2) \quad \ldots\ldots\ldots \quad (20)$$

and

$$\mathrm{C_3} = \mathrm{l_s}\left[1/2 - \frac{1}{12}\,\theta_3 + \frac{m}{\theta_3}(1/2 - 1/4\,\theta_3^2 + \frac{1}{48}\,\theta_3^4)\right]$$

$$+ \mathrm{W}(\sqrt{\theta}_s/\sqrt{\theta_3})(\sin\theta_3 + m\cos\theta_3) \quad \ldots\ldots\ldots \quad (21)$$

As values θ_1, θ_2, θ_e in turn are assigned for θ, the corresponding x's and y's are x_1, y_1; x_2, y_2; x_3, y_3.

For a further simplification, let Eqs. 2 represent the x's and y's in terms of special values of θ, as indicated by the subscripts.

Solutions by iteration of the following equations will give the desired points of intersection 1, 2' and 3':

$$K\,C_1 = l_s\,(y_1 + m\,x_1) - (\sqrt{\theta_s}/\sqrt{\theta_1})\;x\;m\;a \qquad (22)$$

in which $\theta = (\theta_1 - K)$ = recurring value of θ_1;

$$K\,C_2 = l_s(y_2 + m\,x_2) - (\sqrt{\theta_s}/\sqrt{\theta_2})\,[W(\cos\theta_2 - m\sin\theta_2) + m\,a] \qquad (23)$$

in which $\theta = (\theta_2 - K)$ = recurring value of θ_2; and

$$K\,C_3 = l_s\,(y_3 + m\,x_3) - (\sqrt{\theta_s}/\sqrt{\theta_3})\;[W\,(m\sin\theta_3 - \cos\theta_3) + m\,a] \qquad (24)$$

in which $\theta = (\theta_3 - K)$ = recurring value of θ_3.

Preliminary approximations for the θ's in applying Eqs. 22, 23, and 24 are as follows:

$$\theta_1 = (l_1/l_s)^2\;\theta_s \qquad (25)$$

$$l_1 = a - (a/l_s)^3\;y_s\cot\alpha \qquad (26)$$

where

$$y_s = l_s\left(\frac{1}{3}\,\theta_s - \frac{1}{42}\,\theta_s^3 + \frac{1}{1320}\,\theta_s^5\right) \qquad (27)$$

and

$$\theta_2 = (l_2/l_s)^2\;\theta_s \qquad (28)$$

where

$$l_2 = l_1 + W\cot(\alpha + \theta_1) \qquad (29)$$

and

$$\theta_3 = (l_3/l_s)^2\;\theta_s \qquad (30)$$

where

$$l_3 = l_1 - W\cot(\alpha + \theta_1) \qquad (31)$$

As each point of a bridge pier is determined, its adjacent distances to the previously computed points may be compared with the results obtained by the use of Eq. 32, where l_o and l_i are the arc lengths of the outer and inner curves from the TS to any point radially opposite points 2 (or 3) on the base spiral:

$$l_o = l_2 + (l_2/l_s)^2 \; W \, \theta_s \quad \text{(32a)}$$

$$l_i = l_3 - (l_3/l_s)^2 \; W \, \theta_s \quad \text{(32b)}$$

in which θ_s is in radians (multiply by 0.017453, if in degrees).

Example: Given l_s = 300 ft, $\theta_s = 9°$, W = 15 ft, m = 2 ($\alpha = 63° \; 26.3'$), and a = 160 ft. Required: (1) x_1, y_1; (2) x_2', y_2'; (3) x_3', y_3'. Note: Because of limited space, the solution will not be carried beyond the 2nd approximation for the θ's. A summary of the results is given below:

(1) $l_1 = 158.81$; $\theta_1 = 2° \; 31.3'$; $C_1 = 6958.32$; $K = -0.0000304$(radians)

$l_1 = 158.86$; $\theta_1 = 2° \; 31.44'$; $x_1 = 158.8291$; $y_1 = -2.3324$

(2) $l_2 = 165.55$; $\theta_2 = 2° \; 44.4'$; $C_2 = 6473.76$; $K = +0.00003198$(radians)

$l_2 = 165.49$; $\theta_2 = 2° \; 44.3'$; $x_2' = 166.1688$; $y_2' = 12.3469$

(3) $l_3 = 152.12$; $\theta_3 = 2° \; 18.8'$; $C_3 = 7632.36$; $K = -0.0000004$(radians)

$l_3 = 152.12$; $\theta_3 = 2° \; 18.84'$; $x_3' = 151.4896$; $y_3' = -17.0355$

CHECK: $m = \dfrac{y_2' - y_3'}{x_2' - x_3'} = \dfrac{29.3824}{14.6792} = 2.0016$ (exact value = 2).

Journal of the
SURVEYING AND MAPPING DIVISION
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HIGHWAY LOCATION AND DESIGN BY PHOTOGRAMMETRIC-ELECTRONIC COMPUTER[a]

By S. E. Ridge,[1] F. ASCE

SYNOPSIS

By combining photogrammetric techniques and electronic computers, the engineer is able to locate and design highways with economy and speed. Such equipment allows for a thorough study of the many alternatives available in highway design.

The integration of aerial photogrammetry and electronic computation into highway engineering provides the design engineer with powerful tools, which not only enable him to increase his productivity but also make it possible for him to improve the economy of his design. When used properly, these tools provide the design engineer with more information, more accurate information and provide that information in much less time than was ever before possible. Aerial photography, aerial photogrammetry, and electronic computation are of value in all phases of highway location, design, and construction; the area-study, the location-study, the final design, the preparation of the engineer's estimate and the final payment computation. They, at the same time, provide the highway engineer with material that is extremely valuable in giving the general public an understanding of his decisions in regard to route location.

Before going into detail, a brief mention should be made of the real purpose behind the effort to promote the adoption of these methods. Highway engineers are, of course, interested in saving engineering costs. Every dollar saved can

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[a] Presented at the October, 1958 ASCE Convention in New York, N. Y.

[1] Ass't Chf. Div. of Development, Office of Operations, Bur. of Pub. Rds., Washington, D. C.

be used for highway construction and the highway engineer's purpose is to build highways. Even more important, however, is the time saved. The states and the federal government have authorized the expenditure of highway funds at a rate far in excess of anything approached heretofore. This is all to the good, but these funds, as such, are of little value to the motorist. To be of value, they must be translated into in-place highway construction. Engineering is the first step in this translation process.

The construction cannot start until the engineering is complete. Past experience has shown that the engineering process, from fund authorization to contract letting, has in many cases taken as much as 2 yr. An article[2] has shown how far we have gone in reducing this time lag. Only 27 weeks was required to move the project from authorization to contract. Six for preliminary line location, sixteen for design, and five for advertising and letting. This is far less than the 2 yr formerly required.

This is not, however, the most important reason for using these new tools and techniques. It is far more important to improve engineering. Engineers are professionals and as professionals, they should be more concerned about the quality of the work than the speed or cost of it. In addition, far greater savings in time and funds can be effected by performing a better location and design job than by performing that location and design job faster and cheaper.

A better location and design job means many things. It means the design of a project that can be constructed at a lower cost. A 1% or 2% reduction in construction costs is far more substantial than a similar reduction in engineering costs. It means a project that is a tenth of a mile shorter. A tenth of a mile reduction in the travel of the volume of traffic that will use a portion of the interstate system in the next 20 yr means an enormous saving of man hours and transportation costs. A better design can also mean a 1/2% reduction in the steepness of a grade for a mile or so. This could result in far greater savings of time and money than would the complete elimination of engineering.

Highway engineers are, therefore, using these new devices in order to improve their engineering analysis. By so doing, they can be of much greater service to their ultimate client; the highway user and the taxpayer.

The first step in highway location is the area-study. In the area-study phase, the entire area between the points of origin and destination is studied to ascertain the most acceptable corridor or corridors. High accuracies and excessive details are not necessary. In many cases, existing maps and photography will suffice for this study. If no photography is available or the available photography is too old to show the current development of the area, a semi-controlled mosaic made from photography of a scale of 1 in. to 2,000 ft should be acceptable. The photo-contour map is an excellent tool. Using this data, several alternate lines are studied in regard to probable traffic use, transport costs, construction costs, right-of-way costs, potential development of the area, etc., and the most suitable corridor or corridors are selected.

Where vertical control within the areas under study is not available, the airborn profile recorder (APR) is a most valuable instrument. The APR measures, by means of a radar beam, the distance between an aircraft and the ground, and translates this to actual elevation by means of a sensitive pressure altimeter. The horizontal position of the line is determined from synchronized air photos. With some control at each terminus of the flight line, this provides a very acceptable center-line profile for this stage of location. Such a profile,

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[2] "Eleven Miles of Interstate Designed in 16 Weeks."

or profiles, also enables the photogrammetrist to ascertain the horizontal scale of the photography.

After the area-study has been completed, the next phase, the location-study, is begun. As the corridor of the proposed location has been reduced to a width of 1 mile or $1\frac{1}{2}$ miles, the photography for this phase can be taken at a scale of 1 in. equals 1,000 ft. This photography is usually a single strip exposed at 6,000 ft above the ground and covering a width of 9,000 ft. Another mosaic is prepared at a scale of 1 in. to 1,000 ft and a set of prints are made for stereo comparison. The necessary ground control for this photography (usually four points per photograph) is normally obtained by use of United States Geological Survey data extended into the flight strip by ground survey.

However, the first-order instruments now in use combined with the electronic computer can be used to bridge fairly long lengths of such photography. With good control at both ends of the flight strip, and some in the center, the first-order instruments can extend the control through the flight strip and check on the control existing at the other end. The computer can then distribute the error over the flight strip. Proper use of this method will produce ground-control throughout the flight strip acceptable for this phase of location.

The photography is used to study the soil existent along the lines, development of the area, drainage patterns and to locate aggregate sources, etc. The stereo model is used to study the ground configuration.

At the present time, there are two general methods of analyzing the topography. One is the preparation of a 1 in. equals 200 ft topographic map and the use of this map to provide ground elevation data on the several selected center lines. The center lines are laid out on the topographic map and the cross-sections are obtained by scaling the distance to the contours. These data are then analyzed in the electronic computer to produce the earthwork movement required for the selected grade lines. The use of the electronic computer allows the engineer to study far more lines and grades than was previously possible by hand methods.

Ordinarily, the electronic computer operation consists of several steps. The engineer selects the grades and vertical curve characteristics and the computer develops the center-line elevation at each cross-section station. These data together with the proposed cross-section of the highway are then fed into the computer to calculate the earthwork movement. Super-elevation and other refinements of design are not usually used at this stage.

A second method of analysis eliminates the need for a contour map at this stage by substituting for it a planimetric map and what is called a digital terrain model (DTM). The DTM is, in essence, a series of cross-sections eminating from a baseline roughly parallel to the center line at all points.

From an examination of the model, the engineer selects the center line he proposes to use and the horizontal curvature. This data is fed into the computer and the computer produces the stationing of each line of terrain data on the selected alinement, the skew angle between each line of terrain data and the center line, and a profile of the selected line. The computer can also produce profiles at any selected distance to the right or left, at the selected line.

The engineer then studies these profiles, which can be produced mechanically in the line plotter, and selects the stationing and elevation of the intersects of the vertical alinement and the vertical curve configuration he wishes to use. The electronic computer establishes the elevation of the trial grade line at each intersect with the terrain data.

These data together with the basic cross-section of the highway, data relative to the slope to use on cuts and fills of various heights, and compaction factors, are then fed into the computer and the earth movement in the form of a mass diagram and data regarding the limits of construction, and the elevation of the slope stakes on each side of the highway are produced. These data can also be mechanically plotted for visual inspection on the line plotter. The main difference in the calculation is that the cross-section areas are computed from skewed cross-sections rather than right angle cross-sections.

The engineer studies the results and makes his decision regarding the changes he desires in the alinement and/or grade, and the process is repeated until he has obtained an alinement that provides the best possible economy considering construction costs, right-of-way costs, transportation costs, and the general economic impact of the construction on the adjacent area.

Further development of this method is underway. It should be relatively easy to obtain data on right-of-way costs and to develop a program which would produce the approximate cost of obtaining the right-of-way for the selected line.

In addition, a program could be developed which, given the estimated traffic, would produce the cost of transportation over the selected line. With this data available to him, the engineer could produce, in a very short time, a very comprehensive analysis of the cost and value of the several lines investigated at the preliminary location phase.

The methods of obtaining these data for use in the DTM system are now fully developed. In 1956, the first device for the automatic translation of the analog information, produced by the photogrammetric instruments into tape or card records, was developed. Today at least five such instruments are in use. Generally, they consist of a manually operated keyboard on which the station of the cross-section or the X ordinate can be entered; a bar that can be set on the location of the X section in the model and which will then automatically register the offset from that line, the Y ordinate; and a device which will translate the vertical movement of the photogrammetric instrument into a true elevation, the Z ordinate. As the instrument is set on a point and adjusted, a button is pushed and the information is punched onto cards or a tape. A commercial model (TDT) is now available.

In addition, a device known as AUSCOR has recently been developed. This device automatically brings the images produced by the photographic pairs in the stereoplotter into focus. It has two principal uses. It can be used to automatically adjust the photographic pairs in the stereoplotter. This operation formerly required considerable time. The AUSCOR now performs this operation in a matter of minutes. After the photographic pairs are properly oriented in the photogrammetric instrument, AUSCOR automatically operates the vertical adjustment of the plotter and keeps the images in focus as the head is moved through the model. This enables the engineer to take terrain elevations with much greater speed than was heretofore possible.

With these two devices, the AUSCOR and the TDT, large numbers of terrain elevations can be obtained and recorded in a very short time. This makes it possible to economically record much more detailed terrain information on much larger areas. This increases both the accuracy and scope of the line investigation.

These data can be recorded and stored on cards or paper tape. When a location through a particular area is to be studied, the paper tape or cards can be transferred to magnetic tape or a similar memory device and used in connection with the terrain model system. The use of the magnetic tape is suggested since, with it, the analysis of the selected lines can be accomplished with much greater speed than would be possible if the paper tape or cards were used directly.

CONCLUSIONS

We now have the equipment and the methods necessary for a much more detailed and much faster analysis of possible highway locations at the preliminary location stage. With this equipment and with these methods, it should be possible for the highway engineer to improve his engineering analysis and to thereby produce far better and more economical highways. It is up to the highway engineers to put these new techniques and tools into use and to obtain the benefits possible from their use.

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Journal of the
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THE BUILDER OF THE NEWPORT (RHODE ISLAND) TOWER

By Edward Adams Richardson[1]

SYNOPSIS

The Newport Tower, of archaeological interest, is investigated with regard to structural design and to determine possible reasons for the window and fireplace arrangement. The design proves adequate, by modern standards, for a particular church structure, while the windows and fireplace form a sophisticated signalling and ship guidance system characteristic of the 14th century.

INTRODUCTION

The problem of the Old Stone Tower, or Old Stone Mill at Newport, R. I., has given rise to a very extensive literature. It seems important that some attention be given to the builder of the tower, whoever he may have been, whenever he may have lived. As the tower itself is that man's monument and his work, it is conceivable that the tower itself can be questioned effectively through a search for the knowledge and skill which the builder must have possessed. Studies of the history of the site should be utilized.

As some readers may be unacquainted with the relic, a few words of introduction are in order (Fig. 1). In Touro Park, just south of Mill Street, which leads up from the Jamestown Ferry, there stands a rude structure of rubble masonry in lime mortar. It is not particularly impressive at a casual glance. A substantially cylindrical tower about 23 ft. in outer diameter, about 26 ft. in overall height, the upper part a solid wall with a few window and port

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[1] Publications Dept., Bethlehem Steel Co., Bethlehem, Pa.

openings which is carried, in turn, on semicircular arches, and these, in turn, on eight circular columns. The roof and floors, known to have been of wood, have disappeared. The plaster inside and outside is almost completely gone. Holes intended for large floor beams occur at about the level of springing of the arches; others for a second floor may be seen. On the east side there is an old fireplace built into the wall. Two flue holes proceed upwards from the

FIG. 1.—THE OLD STONE TOWER AS AT PRESENT, LOOKING EAST.

corners thereof inside the wall to discharge through small ports in the outer wall surface. Evidence of a stairway from one floor to another may be seen, together with various slots for table tops and cubby-holes for closet purposes. The ground at the base of the tower has an elevation approximately 84 ft. above the mean sea level.

The tower itself once had views around much of the horizon. The columns must have been intended to carry at least the roof of a one-story encircling structure, for the outer sides thereof have caps adapted to such use and the arch load is imposed inside the column center lines. Further details will be supplied as they are needed.

HISTORICAL BACKGROUND

Those desiring to study the various hypotheses regarding the tower and its possible origin may read profitably and with enjoyment the four books which follow. These, in turn, list extensive references to other books and papers. A brief study may be found in "The Old Stone Mill" (1955) by Herbert Olin Brigham. Philip Ainsworth Means has written a comprehensive historical study and discussed various hypotheses in "Newport Tower" (1942). A very interesting development of the Viking hypothesis can be found in "The Lost Discovery" by Frederick J. Pohl (1952). "America 1355 - 1364" (1946), by Hjalmar R. Holand, develops this hypothesis for a particular period.

The actual recorded history of the tower is remarkably slight. Between 1629 and 1633 William Wood was in America. Returning to England, he published a map of eastern New England in 1634. At the present site of Newport he wrote, "Old Plymouth." Presumably ruins of an earlier settlement were visible there. It may be a reference to the existence of the stone tower. Now Sir Edmund Plowden petitioned for a patent to cover all of Long Island and part of the mainland in which mention is made of a "rownd stone towre" some distance from Long Island (east end) by sea. No one has suggested the existence of more than one tower, and most have not suggested it, so it would appear the Newport Tower was then standing and could be used. The Plowden petition for the New Albion patent was acted upon about 1632 by the king. When William Coddington led the colonists to Newport in 1639, it is stated that he saw the tower and endeavored to learn from the Indians their traditions concerning it. They had none. Our authority for this is B. J. Lossing, who, circa 1848, talked with Governor William C. Gibbs, then its owner, who passed on to him the tales handed down from the days of Coddington.

Assuming the probity of Lossing and of Gibbs, is it reasonable to suppose that a reliable story could be handed down for nearly 175 years? The writer must invoke the history of his own family to show that it is possible. His mother, then not over twelve, listened to the stories of the Revolution as told by Samuel Whittemore, her grandfather, concerning the acts of his great-great-great-grandfather on April 19, 1775. They related that he had chased the red-coats down a lane and killed them. The writer was told about these stories by his mother. She had thought these tales merely the tall stories a beloved grandfather liked to invent, so she had never learned them well enough to recount them in their entirety. It is now 200 yr since the events mentioned, just as the stories heard by Gibbs would have been nearly 209 yr old, at most, when repeated to Lossing. If you should visit Arlington, Mass., and go a block east from the square to a school grounds, you would find a stone reading substantially as follows: "Near this spot, on the 19th of April, 1775, Samuel Whittemore, then eighty years old, shot and killed three British soldiers. He was shot, beaten, bayoneted and left for dead. He recovered and lived to be ninety six years old." It would have been a little difficult for the relater of the stories to make them any taller than the historical facts. At least the historians placing the monument are being given credit for reliability, in spite of many known instances in which historical societies have erred in the placing of monuments. A minor example of error was noted by George A. Richardson, the writer's brother. In checking the course of the Sullivan Trail, he found the road wound through the valleys of Easton, Pa., avoiding the hill now crowned by Lafayette College. The data comprised the pacings of distance by the military engineer of the expedition. This did not keep the local historical society from placing a road marker near the top of the hill, however. We must conclude that the story, retold by Lossing, placing the tower in existence as early as 1639 and sufficiently early to enable the Indians to forget all about its origin, cannot be ignored.

The first definite mentions of the tower occur in 1677. The more important of these is found in the will of Governor Benedict Arnold, bequeathing—"my Stone Built Wind-Mill"—. Since there is a record dating from August, 1675, to the effect that the windmill built and operated by Peter Easton had been destroyed by a storm, there had arisen the sudden public need for replacement. Apparently this was furnished, at least through the conversion of the Old Stone Tower, by Arnold. It is to be presumed from this statement that the windmill belonged to Arnold and that at least part of it was of stone. If the writer were to bequeath his home, he might write —"my frame-, and brick-built house"—. Nothing more would be implied (although his wife might well question the —"my"—).

There is remarkable little in the records concerning the tower during the years which followed, when practically no one denies that the tower was there. Yet lack of mention of the tower earlier is used seriously as an argument by historians to prove not only that the tower was not there but that Arnold built it. There is ample material extant to prove that the writer did not build his present home. In fact, it was started about 1856 and finished about 1870 or later. It is probable that the writer, though he may have seen this house many times during 33 yr, did not mention it earlier than 5 yr ago. It was purchased about 3 yr ago. Let it be supposed that a hundred years pass during which heirs, rag men, fire, war and the like produce such an attrition in written records that only a will such as that suggested should remain. Conditions become comparable with those relating to tower records. If the same method of argument should be used in this case as certain historians have used in that of the tower, it would then follow that the present writer's house must have been built by the writer, in spite of all inherent characteristics showing that it must have been built earlier.

At this point it seems well to consider a few of the many reasons why no previous mention of the tower has been found. The remark relative to heirs, rag men, fires, etc., obviously must be listed. A brief reference to "a stone ruin" would not keep a letter from being destroyed when the necessity of getting rid of old papers arose. In the year 1639 it may be assumed that many of the colonists were illiterate; hence the frequency of writing would not be great. Even partial literacy does not encourage long letters, or those which deal with anything excepting matters of importance to the writer. Besides, colonists as a class are a people engaged in a continual battle to maintain even a marginal subsistence. This means that they must work as hard as they can to keep alive and must make some slight provisions for better times. Under such conditions even the literate people will be limited, in their written notes, to the more pressing matters.

Consider the behavior of a group of colonists arriving in a new land. Their leader will tend to take such notes of odd and unusual features as may deserve mention, while the other members will tend to disregard everything not directly related to securing a desirable home and lands. Hearsay testimony is available that William Coddington did note the tower and made a few relevant inquiries, although he may not have felt that it was worth recording. Aside from their natural concern with the pressing business of keeping alive, most of the colonists came from a country where ruins of all sorts, from a hump in the ground to a ruined castle or cathedral, were normal parts of the landscape. Being accustomed to the large and striking ruins, a small tower would be quite unimpressive. Besides, these same colonists might argue, men had been crossing the Atlantic for nearly 150 yr, so why be surprised if some of them built a tower. They knew full well that before the Pilgrims reached America the boats of the fishermen in ever increasing numbers were going west to the coast of Maine each year. They had been surprised to have Squanto speak to them in an English learned from the Popham colonists of Maine, but once that surprise wore off, they expected to see the signs of people who had come earlier.

There is still another consideration. Suppose the men and boys, noting the tower, thought of the freebooters of the Spanish Main, and conceived the idea that some of them had established a strong point here and might have hidden treasure. This tower now stood on the land of one of their number. In order to dig for treasure, an agreement would have to be drawn, granting the owner a portion of anything uncovered. In any case the owner would insist that the property should be returned to its original condition after digging. As farmers, the colonists would be required to keep the top soil separate from the hardpan below. So they would do a very thorough job of digging up the site and would find nothing. The normal artifacts of construction are scattered, any old metal is saved for such use as might be found, and the new artifacts of the diggers get into the back-fill. Rocks are loosened and removed from below columns to search for secret hiding places. Considering the work on their own properties that must have remained undone during this community dig, it seems certain that the good ladies had much to say to their men folk that would not be considered complimentary. Men are not noted for publicizing their errors and follies. The ladies, granting them literacy, do not care to publicize the shortcomings of their mates, for that might reflect upon them personally. The tower would be a most unpopular subject, so, the less said about it the better. By the time leisure to study the structure became available, most common knowledge would be gone.

Much has been made of the failure to find a stone tower as an asset in the listings for taxes. Suppose a man finds his land encumbered with a stone tower for which he can find no use, one too big to tear down, yet taking up valuable space. Human nature does not call such white elephants to the attention of the tax gatherer so that one may lose even more through the paying of taxes on the useless property. It would have been surprising if the tower had been mentioned in such lists before 1675. Even a governor must have decent respect for the interests of the colonists in drawing up such lists. Of course the owner is inclined to brag a bit when his own ability to recognize and seize an opportunity enables him to put his white elephant into a highly profitable business and at the same time gain the plaudits of his neighbors for his care for the public weal. He can now afford to pay the taxes. If mention of a tower in a letter to England might hurt a neighbor by getting such a structure on the tax lists, there would be good reason for not putting anything concerning the tower in letters written to England or elsewhere.

It would not be very difficult to frame other reasons why the tower would not be mentioned. Colonists, by and large, may tend to lack curiosity; business is so pressing there is no time for the day-dreamer, the poet, the artist or the student of anthropology. Kill the Indians, kill the animals; why waste time in learning about them? Before such things can receive proper attention, there must be leisure.

At least one archaeologist has investigated the ground inside the Newport tower. During 1948 and 1949, William S. Godfrey was commissioned to do the work. Mr. Godfrey hoped to find Viking relics. The report of these investigations contains some interesting points. Under one of the columns there was quite a cavity that no competent engineer or builder would leave. It suggested treasure hunting at some past period. There were no signs of any Viking artifacts; in fact, strange as it might seem, the usual chipped stones and other debris of a construction site were missing. Godfrey classified the site as "undisturbed"; in other words, after the time of the builder, no one had dug there. But this finding was based on lack of admixture of top soil and hardpan below. Such artifacts as were found were those typical of the 17th century colonists. Coins dating from as early as 1696 were found. According to the usual mode of interpretation of data, the lack of normal construction debris should have indicated that the tower had never been built. This is a bit difficult to admit as the tower is certainly there. But according to the coins, the tower could not have been built before the first years of the 18th century; 1696 is nearly 20 yr later than 1675. If reference is made to the present writer's hypothesis that the site was dug up in the search for treasure, it will be seen that many of the oddities of the findings made by Mr. Godfrey would be normal to such a "disturbed" site. The present writer finds the "certainties" of Mr. Godfrey unwarranted by the findings as reported.

The purpose of the writer in these attacks upon the validity of the conclusions of others is to maintain open hypotheses regarding the origin which would otherwise be unavailable for use. The writer does not feel that he is unduly unfair to those who have worked in this field. One noted anthropologist was told that the small openings on the second floor of the tower were for signal-receiving purposes. Hie reply amounted to saying that this was certainly an unwarranted point of view. Obviously if a person wished to see, he would make the windows as large as possible so as to see better. This seemed to the writer rather an odd conclusion from an anthropologist, for such a person would know that the pupil of the human eye is only about 2/5 in. in diameter in the dark, so that a suitable hole at least that big would insure perfect vision. Besides, and this point was completely ignored, the viewer might find that enemy missiles were more dangerous than certain sight limitations. Besides it is possible the builder had no glass for such things. In that case, and in winter, the smaller the windows the better, provided they would serve their purpose otherwise. In view of such criticisms, showing a lack of what we like to consider "common sense," it becomes necessary to emphasize the writer's point of view.

If there is reason to suppose that the colonists of the 17th century did not build the tower but rather some one much earlier, the range of hypothesizing broadens. Frederick Pohl lists distant voyages on the Atlantic from early times until 1492 which suggest that many peoples have records of trips. Few of these indicate a stop long enough to permit the building of a tower. Only Holand offers a period of about 9 yr, which would be sufficient for such an undertaking.

Why should the present writer be so anxious to keep a wide range of hypotheses open? Although he considers the Arnold hypothesis unlikely, it has not been rejected; the only real rejections are of certain highly illogical conclusions drawn from such data as are available. The history of science is replete with the shifts back and forth between one reasonable hypothesis and another. To be sure slight changes may occur in "the same" hypothesis between the time it becomes ruling and the next, and such changes must be expected as knowledge increases. The failures of science have tended to occur when a reasonable hypothesis has been "proven" untenable by some mockery of logical thought. Those interested in securing a better understanding might well read, "In Search of Adam," by Herbert Wendt, a book dealing with the historical shifts in hypothesis, the great reigning minds refusing to consider objections, and persecuting those who held new points of view in conflict with "orthodoxy." It is not a pretty picture of the scientific mind in operation, but it is an excellent one of human beings swayed by petty emotions. A particular example of oscillations between hypotheses is related to spontaneous generation. Years ago, every one "knew" that maggots and germs developed spontaneously in putrid matter. Then Louis Pasteur came along and "proved" that spontaneous generation is impossible, thereby believing he had proven the fixity of species. Much later Harold Urey came along, performed his famous experiment using electric discharges in an atmosphere of ammonia, carbon dioxide and water vapor in the presence of water and was able to show that a wide range of organic compounds were formed thereby; hence the motion of spontaneous generation of life became a distinct probability. The regeneration of the earth's atmosphere to one primarily of nitrogen and oxygen by plant life could be shown to be probable. This is but one of a large number of examples.

BUILDING OF THE TOWER

Those who mapped the tower noted that the north and south columns had their centers in the meridian. It becomes of interest to see how this might be done. Most writers say that the north star, Polaris, also known as α-alpha Ursae Minoris would serve. They do not note that, though the distance of this star from the pole of the heavens is only 56' of arc this year, it was about 2°31' away in 1675 and nearly 4°43' in 1358 (600 yr ago). In no case is a single direct reading possible unless the observer has a modern transit instrument, a good chronometer properly set, and a copy of The American Ephemeris and Nautical Almanac. A simple method consists in taking numerous sights on this star during some 18 hr of a long winter night so as to determine the horizontal angle to it when furthest west (west elongation) and when furthest east (east elongation). Given the angle between these two extreme sights, that angle may be bisected and the line of bisection lies in the meridian. A somewhat shorter method might be used today to locate the meridian: as the stars change position with time, it might not apply in 1675 or 1358. δ Cassiopeiae had a right ascension of 1 hr 20' 53.69", and ζ Ursae Majoris had a right ascension 20'54.62" in 1925. A straight line between them would pass very close to the pole of the heavens. If this straight line were vertical at the instant of observation, it would determine the meridian within a few seconds of arc. For more precise work, this line when vertical would serve as a time origin from which to measure about 13' of time until Polaris were at upper or lower culmination and on the meridian. The time for observation could be shortened very materially if such a choice were available to the observer. It would be unreasonable to suppose the quicker method would be known to the builder, hence the long method would be forced upon him with all its difficulties and discomforts.

If the builder had knowledge of the making and placing of sun-dials, a much more comfortable method would be available. The sun dial depends upon the sun itself and is most conveniently used about June 1st, although its precision might be adequate for the purpose at any time. First set up a table with a top as level as possible. Near the south edge of the top and about the center of the width make a point on the surface. With this point as a center, scribe several circular arcs between table edges. Support a plumb line so that the line intersects the table top at this center point. On this line and a suitable distance above the table place a knot or very small bead. The equipment is ready. On a bright sunny day, as soon as the shadow of the bead is on the table, start marking its position with care at frequent intervals. Do this until the shadow of the bead leaves the table. Now draw a smooth curve accurately through the points so marked. This curve will cross each arc in two points. The chord of any arc between the points of crossing may be bisected. These chordal bisections and the center point of the arcs will all lie on the meridian of the place within about 1' or 2' of arc.

To a builder who will probably start his job in the spring, the last method will be preferred. Besides, as will be seen later, such a table top can serve many purposes. If established at the center of the tower site, the tower can be laid down to scale on this surface and necessary building lines can be taken off directly. Furthermore, it forms an excellent surface upon which to lay out the window openings. But first the bearings of various landmarks must be obtained and perhaps the elevations of some will be required.

For the establishment of bearings, a simple instrument can be set up very easily. First, prepare very carefully a straight edge. This may be 3 ft long. At one end, erect a thin sheet of wood with a tiny eye aperture. It would be desirable to have this opening exactly over the edge of the straight edge and at least 3 in. above its top. At the other end of the rule and preferably in the vertical plane through the working edge, establish a truly vertical line comprising a thread. Once established, it can be fastened to a suitable frame giving a clear opening about 4 in. or 5 in. high. The alidade is now ready for use. Line up the straight edge on the tower center of the plan. Sight on the desired landmark. When both tower center and the landmark are located, the bearing line for the landmark can be drawn. With a peep hole of the order of 1/16 in. in diameter and a thread of the order of 0.01 in. in diameter, bearings good to about 1' of arc may be taken. All necessary bearing lines can be set down on the table top plan.

Vertical angles may be required to certain points. The alidade could be adapted to such very easily. A movable frame with a perfectly horizontal thread is required crossing the vertical thread at that end. A level of any sort (a trough with water in it and floating wood blocks with pins) can be used to establish the point at which the horizontal thread is exactly at the height of the peep hole. By moving the thread up and down, and measuring the movement, the dip angle may be determined. In this way the builder could determine the dip to water level about 1600 ft away. Military pacing of the slope, carried out with care, plus this angle, would yield the table-top elevation within about 1 ft.

It is easy to show that vertical angles could be determined about as well as horizontal angles, and with about the same precision. The accuracy of the device is adequate to measure the tip to the horizon. The tower horizon distance is of the order of 11-1/2 miles. The horizon dip is about 5/6 of 1' per mile. The actual dip is reduced by about 1/7 by refraction. Hence the actual dip of about 9-1/2' would be reduced to about 8' of arc. If these facts were not known to the builder, he might set up his alidade for vertical angles by sighting on the horizon line on a very clear day, and so have his dips nearly 8' too high. It is not obvious that this can be proven through the tower itself. It is not even obvious that the builder would feel the need of such equipment for vertical angles. The considerable precision with which the tower windows were located suggests some such apparatus was used in laying out the tower, or at least might have been used. Of course a careful surveyor would realize that to adjust his level to the horizontal, he must reverse his instrument and sight of the same horizon. If he did so, he would find a change of 16' in the horizon location.

All that has been shown so far is that an engineer could easily acquire the required instruments for finding the meridian and for the accurate laying out of the tower and its windows from a plan established on a table top. By 1358 engineering ability of a high order could be expected of any man who had been trained in church construction. At that time the best of the Gothic cathedrals were being built. Just as in the case of the establishment of the base for the columns of this tower a circular trench was dug to a suitable depth and filled with rock, so in the case of the Gothic cathedrals and other structures from about 1100 A.D. onwards trenches of this sort were dug and filled with rock for the support of walls and the like. It is probable that any time after the Crusades there were men who could construct and locate sun dials. It seems reasonably certain that such knowledge was available by 1300 A.D. Certainly the means for establishing the meridian in the orientation of churches were known throughout the period of Gothic cathedral construction.

DESIGN OF THE TOWER

Before considering the layout of the windows, it seems well to consider the actual design of the tower. For this purpose it is necessary to set down the dimensions which will be used, for the published dimensions are not always in agreement. In general these will be taken from the drawings of Rowe, except for timbers, and those may be taken from the measurements of Mason. Where there is a question relative to the exact dimension to use, the writer has chosen the nearest measure in Norse feet and inches, after the suggestion of Pohl. A Norse foot is said to have been 12.3543 English in. or 1.02953 English ft. Both dimensions have been given in Table 1.

In checking the tower design, weights of stone work will be based on 150 lb per cu. ft. The probable range is from 146 to 153. This value applies to rubble, including some granite, in lime mortar. It will be assumed that the wood is white oak weighing 46 lb per cu. ft. For such wood, the allowable stresses might be about as follows (lacking better information, present-day values will be used): For white oak, the extreme fiber stress in bending may be 1100 lb psi. If shear is of consequence, the allowable value may be taken as 100 psi with the fiber. Bearing across the grain may be taken as 800 psi.

It is of interest to know that allowable compression on such rubble masonry ranges from about 50 to 83 psi with the majority of specifications setting 70 psi. The allowable bearing pressure on hard clay ranges from 3 to 4 tons (short) per sq. ft. These values have been taken from the digests of building codes for United States cities before 1919 as given in Cambria Steel Handbook.

An oak beam 12 in. by 12 in., 18 ft long, uniformly loaded, can support 11,730 lb. A foot width of 2 in. plank supported on 66 in. clear span will carry 194 psf of floor. There are, effectively, 269.7 sq ft of floor. Two beams can carry about 87 psf including the weight of the floor. The weight of the

TABLE 1.—TOWER DIMENSIONS

Item	Description	English Feet	Norse Feet
1	Outer Diameter of Tower	23.16	22' 6"
2	Inner Diameter of Tower	18.53	18' 0"
3	Column Base Diameter	4.29	4' 2"
4	Column Shaft Diameter	3.26	3' 2"
5	Column Bases 3,4,5,7 Datum Above $\frac{1}{2}$' above ground		
6	Ground Height About 84 feet above Mean Sea Level		
7	To top First Floor from Datum	12.01	11' 8"
8	To top Second Floor from Datum	19.56'	19' 0"
9	To top Present Wall from Datum	25.31	
10	Est. Height to Top of Wall above Datum	27.11	26' 4"
11	Bottom First Floor Beams to Top Floor	3.09	3' 0"
12	Second Floor Pads to Top Second Floor	1.20	1' 2"
13	Height to Center of Arches	9.95	9' 8"
14	Estimated Mean Height of Arches	9.40	
15	Column Height to Top Outer Pads from Datum	8.58	8' 4"
16	Column Height to Top First Floor Pads	8.92	8' 8"
17	Height to C.L. First Floor NE Window	16.73	16' 3"
18	do. SE Window	15.44	15' 0"
19	do. W Window	13.91	13' 6"
20	do. N Port	16.13	15' 8"
21	Height to C.L. Second Floor NE Port	21.79	21' 2"
22	do. SE Port	24.97	24' 3"
23	do. W Port	24.19	23' 6"
24	Fireplace Height to C.L. Arch	4.80	4' 8"
25	Height Datum to C.L. Flue Holes	23.25	22' 7"
26	Square Opening between First Floor Beams	5.32	5' 2"
27	Floor Thicknesses 2"		
28	Estimated Size of Floor Beams 12x12 inches.		
29	Clear Spacing of Second Floor Beams	5.15	5' 0"

floor is about 13.8 psf. Hence the live loading is about 73 psf. The total load, floor and live load, amounts to about 23,460 lb, the amount the two beams can carry. The live load corresponds, roughly, to what might be considered desirable for offices on upper floors, or a somewhat low value for places of public assembly. From this analysis, the floors could have been designed for the type of occupancy that might be expected of them. Let the assumed floor and occupancy weight be set at 24,000 lb each, first and second floors.

It is reasonable to suppose that the roof might have been conical with a 45° apex angle. Including roofing, the weight might be taken as 22 psf on 270 sq ft on 270 sq ft or, say, 6000 lb.

It is of interest to note that the segmental area cut out of the column under the outer pads is between 5 and 6 in. high at the center of the segment. The central angle subtended lies between 85° and 94°. It seems well to assume 90°. The ratio of height to radius is 0.2929, the ratio of chord to radius is 1.4142, the ratio of area to the square of the radius is 0.2854. The center of gravity of the segment is approximately 40% of the height above the base. The total column cross section is 1202 sq in. The segment area is 109.2 sq in. The center of gravity of the segment is 16.12 in. from the column center. The remaining area has a center of gravity distance 1.61 in. from the column center. The height of the segment is 5.73 in., the chord thereof is 27.66 in. To have the total loading on the shaft of the column centered, the loading on the outer caps must be 10% (1.61/16.12) of the total load of the tower thereabove. This, of course, assumes that the builder knew how to calculate moments, which it now seems reasonable to do.

The weight of the stone in the tower, neglecting openings and changes in section, from the mean height of the arches to the top of the tower amounts to 402,700 lb on the basis of the dimensions given. Add to this the loads due to two floors and the roof, or 54,000 lb. The total dead and live load becomes 456,700 lb, neglecting any wind or ice loads on the roof. It seems reasonable to assume the total does not exceed 460,000 lb. But if this is true, then the outer pads of the columns were intended to support 10% thereof, or 46,000 lb. It is obvious that each of these pads could take two 12 in. by 12 in. beams. Assuming a snow or other live load of 40 psf and a dead load of about 12 psf, it becomes possible to calculate the size of the circular enclosure one story high. Suppose that 42% of the load is carried on the inner supports, then the total load is 110,000 lb. The diameter of the enclosure becomes about 56.8 ft. It is not unreasonable to believe that the outside diameter of the enclosing wall was intended to be 60 Norse ft, allowing approximately 1 ft 6 in. for wall thickness and pilasters on the inner sides of the wall at beam supports.

It will be noted that the roof beams are about 16 ft long in the clear. Using sixteen such beams, the spacing at the outer wall is about 11.8 ft, or about 11 ft in the clear. The loading for 2 in. planking under these conditions is about 50 psf. It can be shown that a 12 in. by 12 in. beam 200 in. in length with the loading imposed will be safe. The required moment is about 279,000 in.-lb while that available or allowable is 317,000 in.-lb. The weight of such a structure, neglecting roofing, is about 12.2 psf.

It becomes of interest to see what stresses were allowed in the columns. Now the weight of the columns is about 94,200 lb, so the total weight on the shafts amounts to 600,000 lb, including also the loading on the outer pads of the columns. As the cross-sectional area of the eight columns totals 9616 sq. in., the stress amounts to about 62 psi. The code range in 1919 was from 50 to 90 psi with the majority about 70 psi. There is considerable evidence in this design that the builder was well acquainted with rules for loading his masonry in building a permanent structure.

Now the areas of the eight column bases amounts to 115.6 sq. ft. The loading per square foot applies to the foundation is about 2.6 short tons per sq. ft. From 2 to 5 tons per sq ft would be permitted on hard clay, when the ruling intensity is about 3 tons per sq ft.

This tower appears to have been built and designed by a person well acquainted with stresses to appropriate design requirements and loads suitable for the occupancies contemplated. There is reason to believe that this man might have been trained in the construction of churches, a thing far from impossible at about 1360 A.D. This was not simply a tower built so solidly that practically nothing could cast it down: it was a tower designed to do a specific job and to use the minimum of material for a permanent structure. The designer knew how to balance his eccentric loadings so that the column shafts would be centrally loaded. The size of structure derivable from the assumption that this was done yields a structure of pleasing appearance and of considerable size (Fig. 2). Others have suggested that stone arches might have been used for the support of the roof of the outside structure. This seems highly improbable, for the size of arches required would have produced a roof which would have hidden the windows of the first floor. Using beams as

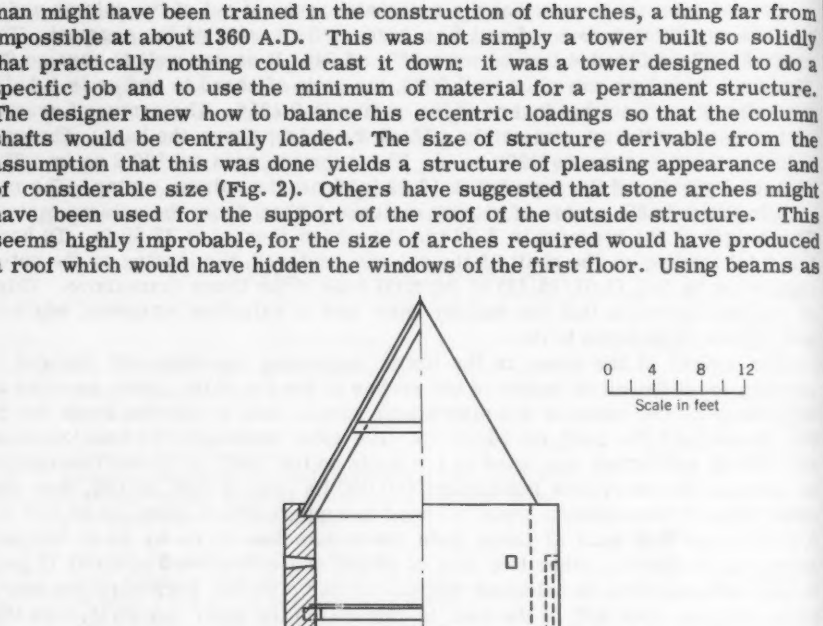

FIG. 2.—SUGGESTED RESTORATION OF NEWPORT TOWER, PARTLY IN SECTION, LOOKING NORTH.

suggested, the roof, either flat or with a slight slope, would have been sufficiently high to keep the stonework of the arches within the first floor of the whole structure, yet would have covered the arches completely from the outside without infringing upon the second floor windows. A suitable outer wall might well have been given a high, perhaps crenelated parapet so that the defenders might protect the roof from direct access by an enemy and safeguard the windows from serving as means of ingress to the tower. Now it is possible, as Holand has suggested, that the west window of the first floor was intended as a temporary ingress into the tower. But with the outer section roofed, this would have been a very clumsy means of entrance. The first floor differs from the second in having a space between the timbers nearly 5 ft 4 in. sq. As there was no stairway to this floor, it seems obvious that a trap door was to be used in this space permitting protected access to that first floor when the enclosure was completed.

In the case of the first floor roof outside the tower and of the tower roof it seems that the roof planking was protected. The tower, with ports from the fireplace close below the roof, could not have an overhanging roof edge which would be in danger of catching fire; that edge had to be protected. The flue holes were only about 7 ft above the top of the fireplace. So, with a good fire on the hearth, flames, sparks and hot gases might be spewed out.

If this indicates that the builder of the tower was a competent engineer and builder, then it becomes obvious that he would have designed and built a structure suited to his clients. Furthermore, if this engineer knew better designs for fireplaces, it seems reasonable to believe he would have used one in a building showing the character of this stone structure. As he used a fireplace commonly accepted in or before 1400 A.D., but very inferior in design and concept in 1675 A.D., it seems necessary to place the construction of the tower near the earlier date. From used to be made of the windows, it would seem essential that he employ the best fireplace obtainable. Certainly nothing about the tower suggests that the client ever instructed the designer that it was to be constructed for flour-milling. The floors are not of the right strength; a fireplace is placed disadvantageously; there are no adequate means for getting machinery into the structure; there is no reason for a surrounding one story structure which might interfere with the use of a mill wheel; and no protection for the flour milling operation is available, and no adequate means for getting meal in and flour out. But if the time came when the tower was needed as a mill, certainly a reasonable adaptation could have been made at a cost, slight, when compared with that of building an entirely new supporting tower.

SIGNALLING

It is now advisable to follow the layout of the structure for signalling purposes. In this case it is necessary to proceed from the final layout to the means used for producing it. The means for setting down the essential data have been suggested.

Many suggested that the windows might have been used for watch tower or signalling purposes. Aside from this article, there appears to have been no published study investigating the nature of such watch and signal means or the significance thereof.

The first step in the investigation comprises determining from cross-sections of the tower through the window openings on the two floors the extreme bearing lines for all openings. Confining attention to the second floor first, since its layout is somewhat simpler, and, in fact, suggests a signal-receiving function. Fig. 3 shows the results of such measurements. Window W5 to the northeast has a seeing range from an azimuth of 7° to 59°. Window W6 to the southeast has a seeing range from an azimuth of 112-1/2° to 162-1/2°. Window W7 to the west has a seeing range from an azimuth of 248° to 285-1/2°. The ports are not big enough for a man to stick his head out, so his range is limited by the cut-off of view as he passes before the opening being used. Of course this arrangement has a distinct advantage in locating the sending signal, particularly if it is a fire at night. Such a station could be accurately plotted on the far wall, partly by the ray at horizontal cut-off of the

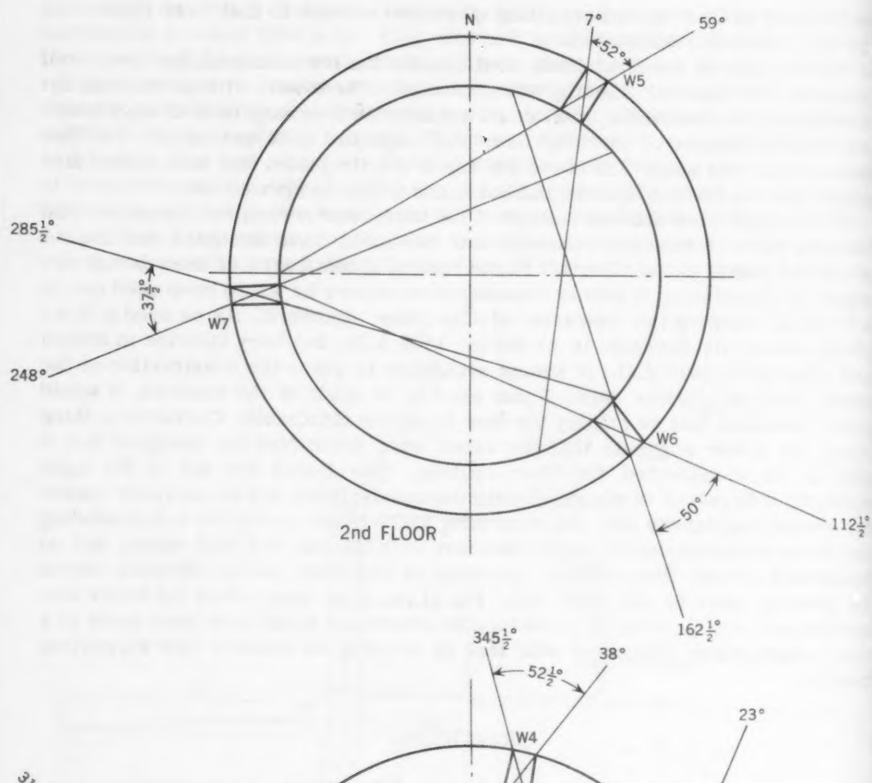

FIG. 3.—FIRST AND SECOND FLOOR WINDOW OPENINGS OF THE NEWPORT TOWER, IN SECTION, SHOWING LIMITING RAYS.

signal, partly by the ray of vertical cut-off by the horizontal edge of the window. If only horizontal cut-off is used, a line on the far wall would identify it. If both cut-offs, a point on the far wall would suffice.

The next step in the study is to lay down these bearings from the tower center upon a chart of the region in order to determine what points might have been of interest to the builder. From such a chart we determine the location of the tower as, approximately, 41°29' 9" N. Lat., 71°18'38" W. Long. In part the Newport, R. I. U. S. Geological Survey Map of 1944, reprinted 1950 was used, in part the U. S. Coast and Geodetic Survey Chart No. 115, Cuttyhunk to th Block Island, reissued August, 1916. As of 1939, the magnetic declination was given as 14-1/2° west of true north while in 1915 is was 13°10' west of true north. In 1915 the declination was increasing at the rate of 6' per yr. Obviously the rate was not maintained. The figure of 15.6° (15°36') shown on Fig. 4 is merely an approximation.

Laying down the bearings so determined upon the Geodetic Chart No. 115, a figure, showing the arrangement of the second floor rays is obtained. From window W5 the whole width of the land approach along Aquidneck Island is covered. There are four possible high spots for signalling in this region visible from the tower. The first is J, or Bliss Hill, El. 148 ft, about 1.5 miles to the north, with a horizon distance of 13 nautical miles. The second, slightly to the west, is K, apparently the point determining the extreme western ray for this window. Station K controls all water approaches from the north and conditions on Conanicut (G) and Prudence (H) Islands. This station is known as Middletown Hill, it is at El. 164 ft, is 3.6 miles from the tower, and has a horizon distance of 13.6 miles (all miles are nautical). Now this station might serve to relay messages from L, or Turkey Hill, at El. 275 ft and 5.9 miles from the tower. Its horizon distance is 17.6 miles. It is more probable that direct signals were used from L. Now L, M (Slate Hill, El. 265 ft, 3.8 miles from the tower, horizon distance 17.6 miles), N (Howland School Hill, El. 183 ft, 3.2 miles from the tower, horizon distance 14.4 miles) determine the outlines of a ridge blocking the view from points beyond. With a few small outposts, the land approaches to the tower were well protected and danger could be signalled quickly from any controlling high point. Obviously any one of these points might be worth study from the standpoint of locating possible artifacts. The number to be expected is small, of course. Although the eastern limit of window W5 is a bit uncertain, certainly the whole range down to station O (Paradise Rocks, El. 170 ft, 2.3 miles from the tower, 13.9 miles horizon distance) is covered. Station I (Tonomy Hill, El. 150 ft, 1.5 miles distant from the tower, 13.0 miles horizon distance) appears to be outside the range of observation. This is not important as the neighboring point J is within sight.

Window W6 has its extreme northern ray fully determined by BB and CC (West and East Islands south of Sakonnet Point, El. 22 ft, about 5.6 miles from the tower, with 5 miles horizon distance). No ship would approach north of this line because of shoal water. It is of interest to note that HH (Gay Head, Martha's Vineyard, El. 147 ft, 22.9 miles from the tower, 12.9 miles horizon distance) is within the zone of visibility. As the horizon distance of the tower (El. 109 ft) is 11.1 miles, a signal on this hill would be seen as a fire just above the horizon. The distance being 22.9 miles, the sum of the horizon distances of 11.1 plus 12.9 = 24 miles, which is a greater distance and thus insures visibility. Perhaps a better beacon might be H'H' (also Gay Head, Martha's Vineyard, El. 170 ft, 24 miles from the tower, horizon distance 13.9 miles).

A few words might be said at this point concerning "horizon distance" and visibility. Due to the curvature of the earth, a straight line tangent to the surface of the sea at "the horizon" will intersect an elevation at a height proportional to the square of the distance. If the height h is in feet above sea-level, and the distance to the horizon is D_s (statue miles), $D_s = 1.224 \sqrt{h}$. If D_n is for nautical miles of 6080.2 ft), $D_n = 1.0663 \sqrt{h}$. Actually the refraction of the air causes the light to bend around the earth so that a body which should appear to have its top in the horizon will be seen with considerable elevation above the horizon. Correction can be made to the tabular horizon distances by adding about 8% thereto. The actual refraction varies slightly. Now the straight line from an elevation to the horizon dips 60" of arc for each nautical mile the horizon is distant. Due to refraction, however, the apparent dip is reduced to about 51" per nautical mile.

In looking between the high points, the test of mutual visibility is, in the limit, the light ray, between that grazes the horizon. Then the distance between the two elevations cannot exceed the sum of their respective horizon distances. Actual visibility requires that the horizon distances corrected for refraction should be used.

To continue the discussion of W6. The southern extreme ray has no apparent limitation excepting coverage of the southeast sea approaches fairly well towards the south of the tower. Perhaps more information will be gathered from the first-floor windows.

Window W7 has its southern extreme ray so located that it skims the 40 ft elevation ground west of Brenton Cove (site of Fort Adams) and reaches the East Passage channel about 9 ft above the water. It would appear that a ship on course might have its forecastle or poop some 10 ft to 12 ft above the waterline to be certain of being seen from the tower. The distance to such a ship would be 2.45 miles. At night rigging would be difficult to see, though a light on the poop could be picked up easily. The northern extreme ray is not so easy to locate, but it appears to have been set to mark the northern limit of Goat Island (W) in the early days before filling was done. There are two other possibilities, perhaps. There is a hill at C (Wakefield Hill, El. 166 ft, 7.9 miles from the tower, horizon distance 14.0 miles) from which signals could be received. It may have purpose later. A rise at D (Boston Neck, El. 95 ft, 6.3 miles from the tower, 10.4 miles horizon distance) does not seem important. Either E (Tower Hill, El. 184 ft, 6.8 miles from the tower, horizon distance 14.4 miles) or F (McSparran Hill, El. 237 ft, 6.6 miles from the tower, 16.4 miles horizon distance) might have significance. Of course such points as AA (Southwest Point, Conanicut Island, El. 70 ft, 3.0 miles to the tower, horizon distance 8.9 miles), V (The Dumplings, El. 25 ft, 2.1 miles from the tower, 5.3 miles horizon distance) and Goat Island W (El. O 15 ft, 0.8 miles from the tower) might be of possible interest as signalling points; but they are more important as points to be watched.

The above covers the layout of the signal-receiving second floor. The builder was greatly interested in three things: the land approaches and possible outposts and beacon on Aquidneck Island; the sea approaches from the southeast; and the channel windings to the west. He was not interested in the sea approaches from the southwest so far as this second-floor receiving center is concerned.

It is possible the southwest approaches were watched otherwise, as by an outpost on Block Island, at an azimuth of 214° from the tower at a distance of 18.8 miles to its Beacon Hill at El. 211 ft and 14.5 miles horizon distance. It is possible that C (Wakefield Hill) served as such an outpost. But it seems more probable that direct signalling from Block Island to the second floor of the tower would have been used if there were any reason to expect vessels to approach from the quarter. Since there are no such provisions, it must be considered quite probable that no vessels were expected. If so, then it is to be doubted that this tower was built at any time during the 17th century, for the Dutch were a military threat during most of that period and most certainly a watch would have been kept in that direction.

Suppose, however, that we consider the conditions of Holand's 1355 A.D. colonists or their expedition. In this case there were no ships to be expected from the west. Any ships sent out in search of the West Greenland colonists would have sailed east and would return from that direction. A most likely approach by such ships would be through Vineyard Sound north of Martha's Vineyard, bearing south of Cuttyhunk Island. Any supply ships might be expected to arrive either through this channel, or perhaps from the southeast, south of Martha's Vineyard. There would always be the possibility that enemy or pirate vessels might appear from that direction, particularly any that might follow a supply ship to a promising colony or small port. Nothing else could expected to arrive by sea. Certainly these considerations fit the provisions made by the tower builder much better than any others that might be suggested at present.

With these points in mind, attention can be given to the first floor. If the second floor was, as it appears to have been, a signal-receiving station, it might well be that the first floor was intended as a beacon, or as a signal-sending station. At night a bright fire on the hearth would provide the necessary light. Hence it seems desirable to consider not only the extreme rays through the first floor windows, but also the rays marking the first appearance of direct fire as well as the region in which constant fire light may be seen. Fig. 3, first floor shows the results of such studies, in determining the necessary azimuths of lines. Window W1 to the east has rays with azimuths of 23° and 110°, and direct firelight is not seen. For window W2 to the south, the extreme azimuths are 126°, and 222-1/2°. Now the beginning of direct firelight is at 213°. The west window, W3, has extreme azimuths of 207-1/2°, and 314-1/2°. The beginning or end of direct firelight are lines with azimuths of 259°, and 271-1/4°. Over an angle of about 1 in 20, or nearly 3° at the center of the range, direct firelight remains constant. Window W4 has no direct firelight. Its range is from 345-1/2° to 38°. With these azimuths, reference may be made to Fig. 4 where they are laid down on the chart.

Window W1 appears to be fully determined by bearings. Station L at Turkey Hill (El. 275 ft, 5.9 miles from the tower, horizon distance 17.6 miles) corresponds to the extreme north bearing, while Sakonnet Point Q (El. 45 ft (maximum), 5.5 miles from the tower, 7.1 miles horizon distance) sets the extreme south bearing. Once again the limits of shallow water appear. This time it appears that the northeast window is arranged so as not to be seen when in operation by any approaching ship. All stations such as L, M, N, O (Paradise Rocks, El. 170 ft, 2.3 miles from the tower, 13.9 miles horizon distance) S (Easton Point, El. 145 ft maximum, 1.6 miles from the tower, horizon distance 12.8 miles), R (Sachuest Point, El. 45 ft (maximum) 3.0 miles from

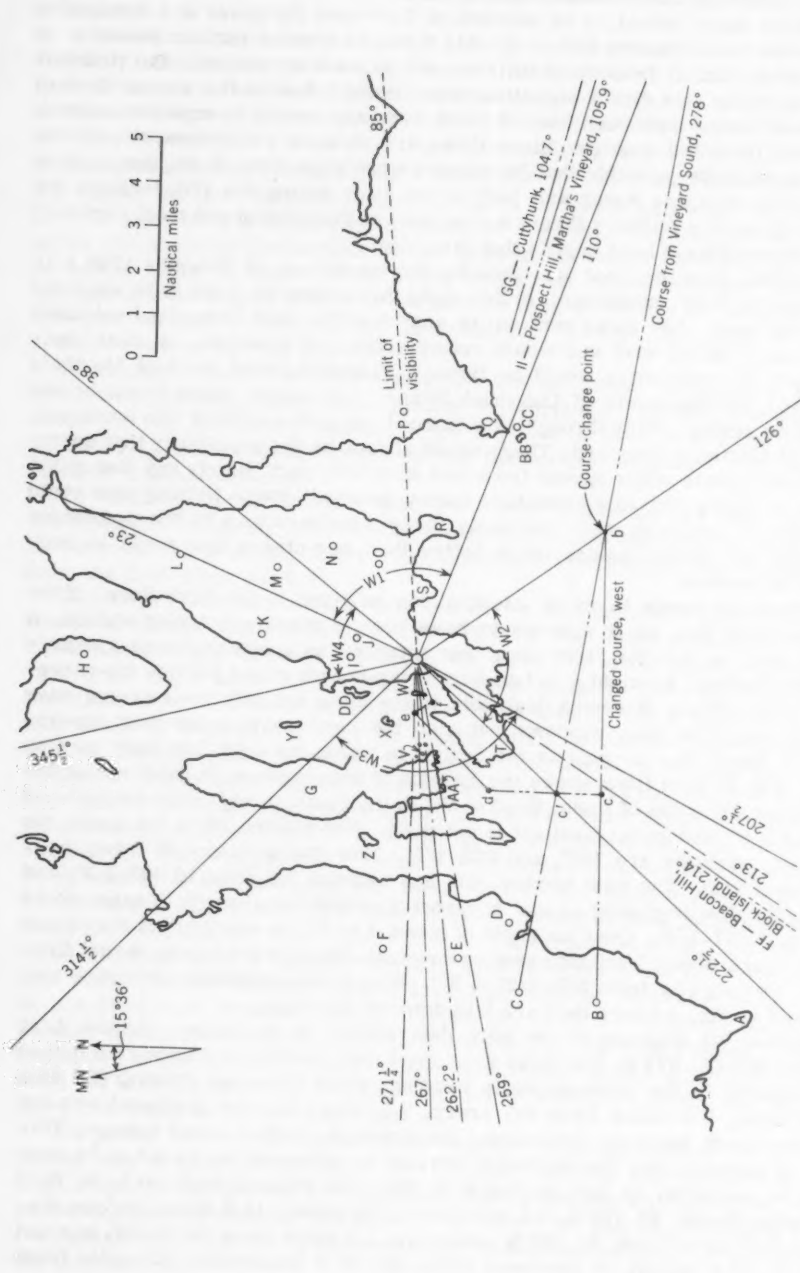

FIG. 4.—FIRST FLOOR WINDOWS OF NEWPORT TOWER SHOWING LIMITING RAYS AND PROPOSED SAILING COURSE.

the tower, 7.1 miles horizon distance) and P (Little Compton, El. 65 ft (maximum), 5.7 miles from the tower, 8.6 miles horizon distance) are within range of the signal. A line with an azimuth of 85° marks the most northerly points not cut off by the ridge L, M, N, O and S. Now it will be seen that Cuttyhunk Island, GG (El. 154 ft, 14.4 miles from the tower, 13.2 miles horizon distance) might see the light from the window, but it is improbable on account of distance. Similarly for Prospect Hill II (Martha's Vineyard, El. 308 ft, 26.3 miles from the tower, 18.6 miles horizon distance).

Departure a lies off the map for a vessel sailing from Vineyard Sound towards Newport. A suitable landmark on a desirable course is needed. Now Wakefield Hill at C (El. 166 ft, 7.9 miles to the tower, 14.0 mile horizon distance) seems like a good possibility. Hence the straight line point of departure a to C. Now a is 14.3 miles from the tower. It is 19.7 miles from Wakefield Hill. Suppose a man's eye aboard a ship is 16 ft above the water. The observer's horizon distance is 4.2 miles. Then, 4.2, plus 14.3 miles is 18.5 miles. Allowing for normal refraction add 8% and the hill top is on the horizon at 20.0 miles for the ship's observer. Hence Wakefield Hill is a very logical landmark, as it will "rise" above the horizon very shortly after the ship takes its departure from a. Hence the straight line as drawn seems like a logical course.

Now it will be see that the extreme ray to the east from the south window, W2, cuts the line aC at b. This could be a course-change point. If so, it seems well to check for a new landmark. Two choices, one behind the other, are available. The first is B, a hill on the Point Judith peninsula (El. 97 ft, tower distance 6.7 miles, horizon distance 10.2 miles. The second is Broad Hill (not shown) (El. of 226 ft, with a horizon distance of 16.0 miles). Broad Hill is 3.74 miles west of B on Point Judith. The distance of course point b from hill B is 11.59 miles. The distance to Broad Hill is 15.33 miles. The observer on the ship would see Broad Hill at 15.6 miles, allowing for refraction, so the hill is visible. Similarly Broad Hill would be on the horizon at 20.2 miles, allowing for refraction. The two hills together may serve as a suitable beacon for a course due west from b.

It will be seen that if there is no change of course at point b, the ship will reach a point where the line of light from the west window is visible, then begins to see direct firelight through the south window, and the direct firelight disappears at c'. This would be the point at which the course should be changed to true north. It lacks a certain degree of safety, since the pilot may not be sure of the point of true cut-off, and thus turn north too soon. Following the course due west from b, a point c is reached, very probably characterized by maximum direct firelight intensity. This seems to be a much safer method for determining the turning point, so it seems very plausible that the intended course is actually a to b to c, then due north.

In passing it may be noted that the direct firelight from the tower does envelop Block Island. The Hill FF (Beacon Hill, El. 211 ft) is 18.8 miles from the tower and has a horizon distance of 14.5 miles. As the tower horizon distance is about 10.4 miles, the sum is 24.9 miles which is a great deal more than the actual distance, even without allowing for refraction. Incidentally, also, a straight line from the tower to FF passes directly through point c. It is doubtful if this would be a matter of coincidences. When available as an outpost, the tower and a beacon on Beacon Hill of Block Island might well be the normal manner of marking the turning point. But the builder appears to

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FIG. 4.—FIRST FLOOR WINDOWS OF NEWPORT TOWER SHOWING LIMITING RAYS AND PROPOSED SAILING COURSE.

have provided for the contingency that the Block Island beacon would be missing, so made final guidance rest primarily on the tower signals alone.

As the pilot proceeds northwards between c and d, he may have occasional glimpses of the west window of the tower, W3. At about point d, he will be able to see the west window wall light over the Fort Adams headland. He could turn due northeast (true) at this point. If a beacon were lighted at I or J, he would head directly towards it, shortly coming in view of the second floor west window. Whether there is a beacon at I or J or not, the builder has provided that the ship can be brought into port. Continuing along the northeast course, he shortly crosses the south line of direct firelight, sees the brightness grow, then become constant, and finally grow less and vanish, leaving only the much fainter wall light. Just as the direct firelight disappears, the course must be changed to southeast (true). The ship recrosses the zones of direct firelight until once again the direct light vanishes, then makes a slow turn to port until the tower light lies directly ahead. Using the tower as a beacon, the ship proceeds straight to port.

This leaves the north window, W4, for consideration. In passing it might be noted that nearly full direct firelight can be seen from E to the west of Narragansett Bay, a place called Tower Hill (El. 184 ft, 6.8 miles from the tower, 14.4 miles horizon distance). This is a very logical point for a lookout, as signals can be received most easily, and return signals can be made from there. It is also about 16.4 miles from Beacon Hill (FF) on Block Island, so is in plain sight of that elevation. As the tower is only 18.8 miles from Block Island, it might seem that direct signalling, tower to Block Island, would be preferred to a roundabout course.

Window W4 to the north seems intended for short-distance signalling only. It is a small port. The beacon points I and J lie at a short distance. More remote locations are H (south end of Prudence Island, El. 175 ft, 7.5 miles from the tower, 14.1 miles horizon distance) also K, L and M. As L and M can receive from the east window, and as M can see both the port and the east window, it is possible this was intentional, and that M was the prime outpost in this direction. Point K is at approximately the same distance, but might have difficulty seeing the north port. The east window is not visible.

The foregoing sums up our knowledge of the window arrangements so far as horizontal angles are concerned. It becomes of some interest to determine the vertical angles to some of these points, as a check. Using window W5, all targets I through O reach the far wall of the tower from 0.55 in. to 1.48 in. below the bottom edge of the window. This allows for earth curvature, refraction, and the like. This window is low, because it can be looked through from the stairs. Otherwise it would be the highest window. The northeast window on the first floor stands about 0.6 ft higher at the center than the first floor north port. But the bottom edges are about the same height. They point to high ground. The southeast window, first floor, is about 1.53 ft (1.50 Norse ft) higher than the west window on center line, but is about 1.29 ft lower than the northeast window. The line of sight on the far wall is about 1.28 in higher than the window sill assuming 3.33 miles from the tower to point B on the course. This is about enough difference in window elevation to have the same portion of the fire visible through the south windows and the west windows. The south window is 2.86 ft high, the west window (clear) about 2.25 ft high. A difference in center height of 1.53 ft makes a difference of about 1.33 ft on the bottom edges of the window. Although not exact on the basis of such calculations, it is obvious that deflections upwards and downwards, as well as the portion of the fire to be exposed, all entered into the locating of the windows. As a result, this writer finds little left to chance. Every dimension received due consideration by the builder for the purpose for which the window was to serve. Just as not all dimensions are critical, but may be left to judgment, so in this case places which do not quite check unquestionably were known to be satisfactory even with modifications for some matter of convenience. A modern engineer makes sure the design is the minimum for doing the work, then makes such changes as facilitate production, or the like, or insure reasonable standardization. This is done without reducing the effectiveness of the device for its intended purpose.

There is one more test to be applied to the plan that has developed during the course of these studies. Is the light sufficient? Will any suggested uses be excluded by such a test? First a few principles must be enumerated. A light source of 1 cp is, in theory, a small, uniformly bright spherical surface, such that at the distance of 1 ft there is a degree of illumination known as 1 ft-c. As the spherical surface 1 ft from the center of the source has an area of 4π sq ft, it is of importance to evaluate the total light emission from the source. This unit is the lumen, a 1 c-p source delivering 1 lm to a square foot of area in setting up an illumination of 1 ft-c. The light intensity on a square foot varies inversely as the square of the distance, since this is the way the spherical area varies with radius, but the total light flow through any spherical surface as a whole is constant, irrespective of the distance. Hence a 1 cp source always emits $4\pi = 12.57$ lm. When light strikes a reflecting surface, the lighted surface varies in brightness with the angle of sighting provided the surface is "matte," but perfectly reflecting. Such a surface of small area illuminated by a source will not appear as bright as 1 cp even when of 1 sq ft area and illuminated by 1 lm. But such a matte surface lining the inside of a tower receives light not only directly, but by nearly perfect reflection from all other tower surfaces. Hence the light through a window may be measured in candle power by the number of lumens of direct illumination. A window of 1 sq ft facing a wall receiving 1 lm per sq ft will behave as a 1 cp source. The direct light from a fire will be of as many candle power as the rate of burning of oily wood is a multiple of the rate of burning of candle flame. Knowing the candle power and the total area, a candle-power per sq ft of front may be determined for the direct fire light.

Having set down the laws of light production and flow, it becomes of importance to discuss the sensitivity of human vision to faint light. Experiment suggests that on a clear, dark night at sea, a white light of 1 cp may be picked up with certainty at 3,200 yd. This is approximately the same thing as saying that such a light at 3,200 yd appears to be as bright as a 3.15 magnitude star from the astronomer's standpoint. The astronomer has set 6.0 magnitude as the brightness of a star which a person can just see when looking directly at it. It may be virtually impossible to pick up otherwise. There are cases where an observer could see a tower signal provided a tube had been set up permanently looking straight towards the tower. The tube limits the field of vision to one easily searched and stared at. It now becomes necessary to estimate the amount of fire-light and the brightness of all windows in terms of candle-power. That done, the distance of pick-up may be set.

The surface of a candle flame emits 3 cp per sq in. of surface. The hearth is such that at maximum a flame surface 2.5 ft sq could be obtained. This is about 650 sq in. At 3 cp per sq in., the fire has nearly 2,000 cp. This is high, so let it be assumed that the maximum does not exceed 1,000 cp, while 1 sq ft of fire seen through a window is about 430 cp. To get this amount of light, oily sticks must be completely and rapidly burned and the products of combustion must be removed. This must be done for several hours. The builder would install the best fireplace of which he might have the knowledge to insure such behavior. He would also make sure the fire would burn uniformly and well regardless of the direction of the wind. In other words, if he knew of what are called "modern" fireplaces he would use them; if he knew of chimneys, he would build one adequate for the purpose of a height greater than that of the peak of his roof. It was essential that he do so. In view of the primitive type of fireplace he did use, it must be considered proven that he knew of nothing better. This means that the tower must have been built before about 1400 A.D. The only possible escape from such a conclusion would be that this was a structure built primarily for eventual use as a church and that church rules and specifications had not advanced to the point of permitting such an innovation. It is well known that sacrificial knives were made of hard stone long after the stone age because no priestly group could bring itself to change the tabus of the past no matter what better material might be available. It seems improbable that hearths would come in this category, but not impossible.

Returning to the intensity of wall illumination, the fire of 1,000 cp emits about 6,300 lm towards the walls and floor and ceiling. The wall area is about 436 sq ft, the floor and ceiling about 539 sq ft. The total area is about 975 sq ft. The lumens per square ft of wall average (over wall, floor and ceiling) about 6.46. In view of the considerable area of floor and ceiling which may be non-reflecting, it is not advisable to assume a luminosity as high as 6.46 cp per sq ft of window facing wall area. A value of 4.5 might not be excessive and has been used. Window W4 with an area of about 0.84 sq ft will be a 3.8-cp source. Such a light can be picked up at 6,200 yd, or about 3 nautical miles. Points I and J can see the port. With some direction, it is possible that some of its light can be seen at K, but it is very unlikely it could be seen at L.

Now window W1 has an area of about 3.73 sq ft, so may rate as 16.8 cp. Its visibility is limited approximately to 13,100 yd or about 6.5 nautical miles. It is probable that K through Q can see this light, although the angle at Q is highly unfavorable, and so is it for L.

Window W2 has an area of about 6.43 sq ft, so may rate as 29.4 cp. It will be visible at 17,200 yd or about 8.5 nautical miles. This is more than ample for the assumed ship's course from A to hill B.

At the point of maximum firelight, the window area is about 1.6 sq ft. With the fire of intensity 432 cp per sq ft, the light amounts to about 690 cp. The actual vertical distance is not as great as the whole window height, so it would be more correct to set the light at, say, 400 cp. This light is good for about 32 nautical miles. The light could be seen with ease at Block Island, yet the ship would be about 4.75 miles or less than 10,000 yd from the tower. At point c, the firelight would appear as bright as a -1.0 magnitude star.

Window W3 is about 2.25 ft square, of 5.06 sq ft in area. Certainly a full 1,000 cp would be available in the direct firelight beam. This would yield a very bright light, relatively speaking. For wall light, this window is of about 22.8 cp, but at point c it would be only about a third of the window, or say 7 cp. The west window might be detected at point c (light visible to 4.2 miles instead of actual 4.75 miles) but could contribute little. There is an interesting question at this distance relative to the ability of an observer on a ship to separate the south window light from that of the west window. It can be shown that the distance is just about close enough to see two lights instead of one.

Two stars about 0.00032 radians apart can be just about separated. The angle here is about 0.00042 radians. So far as can be determined this may have been coincidence; it seems improbable that this might be intentional unless the builder did, as did the writer, make a model test. Point sources of light may be separated at 0.00032 radians, but finite sized lights such as these windows may require nearer to 0.00042 radians. The figuring is so close that it raises interesting questions. The 22.8 cp light is visible to about 15,300 yd, or 7.55 nautical miles. Points F, and perhaps C, could see the wall light. Points such as D, U, Z, G, AA, K, X and W can all see this light. Of course Tower Hill at E is in the line of direct firelight.

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Journal of the

SURVEYING AND MAPPING DIVISION

Proceedings of the American Society of Civil Engineers

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HIGHWAY AND BRIDGE SURVEYS: PRELIMINARY BRIDGE SURVEYS[a]

Closure by the Committee

COMMITTEE ON HIGHWAY AND BRIDGE SURVEYS.—The discussions by Messrs. Rowe, Shockley, Focht, Greer, Reuss, and Zegarra suggest certain additions and deletions that would make the paper more suitable as a part of a proposed ASCE Manual of Engineering Practice.

The Committee accepts Mr. Rowe's suggestion that the scope of the subject matter should be fully defined at the beginning of the paper, and that resulting editorial changes should be made. The definition would include grade-separation bridges. His discussion of additional points to be covered under "Hydrographic Surveys" will be useful in the revision, as will his suggestion that the paper include a discussion of mapping underground utility facilities.

The consensus of Messrs. Shockley, Focht, Greer, Reuss, and Zegarra is that it is not appropriate to include, in a paper on bridge surveys, material on subsurface investigations, which are deemed to lie within the purview of soils engineers. The Committee shall be guided by this opinion. In the proposed manual the Committee will confine the material substantially to the "Location of Test Sites."

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[a] November 1958, by the Committee on Highway and Bridge Surveys.

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STATUS OF SURVEYING AND MAPPING IN THE UNITED STATES[a]

Discussion by Carl F. Meyer

CARL F. MEYER,[1] F. ASCE.—The relatively brief list of "Conclusions and Recommendations" submitted is an excellent distillation of the many parts of the problem examined by the Task Committee. That this committee has been objective and open minded and has considered many viewpoints, is reflected by reading its concise outline of the general background of the problem and the findings of the interim reports.

The overall definition of the field of surveying and mapping and the list of those activities recommended for classification as engineering appear to be consistent with present conditions and shaped to fit future developments.

Those persons responsible for pressuring civil engineering education to continue shrinking surveying instruction cannot with fairness fail to recognize the underlying soundness of the recommendations in Section 3.

Perhaps the most far reaching recommendation is that in Section 4, which is pointed toward the elimination of separate registration for land surveyors. If land surveying is to be defined as engineering (and it should be so defined in order to reflect the increases in complexity of surveying methods, in the scope of auxiliary professional knowledge, and in the economic value of landed property), then the continued bifurcation of registration is no longer necessary. Such separate registration even now detracts from the professional status of both the engineer and the land surveyor.

The recommendation that those who wish to engage in the practice of land surveying and related engineering work should first be required to qualify for a professional engineer license, is a step in up-grading the status of land surveying that should be welcomed by all reputable practicioners in that field.

Naturally, it will be essential for registered professional engineers to "be very careful in judging their own competence to practice property surveying." Such care will require only further extension of the high sense of ethics that presently guides the truly professional man in undertaking work lying only within his special training and experience. The possible fear that the registered engineer who is primarily experienced in land surveying will not also be governed by such professional ethics is not a valid reason for failure to support the Task Committee's recommendation in Section 4. It is in the nature of man to rise to higher levels of moral and professional conduct when given greater public responsibility and the recognition therefor.

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[a] September, 1959, by the Task Committee on Status of Surveying and Mapping.
[1] Prof. of Civ. Engrg., Worcester Poly. Inst., Worcester, Mass.

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SURVEYING IN THE CIVIL ENGINEERING CURRICULUM[a]

By Kenneth S. Curtis,[1] M. ASCE

SYNOPSIS

Current curriculum revisions toward a stronger basic sciences background will mean less emphasis on the art of engineering and the new civil engineer will not be as productive directly upon graduation. However, he will be better equipped to successfully cope with the complex problems of the future and will eventually yield a greater contribution.

The problems in teaching surveying and mapping in the civil engineering curriculum are becoming more involved and complex with the current curriculum revisions which are taking place in our schools of civil engineering in the United States. These revisions are essentially designed to reaffirm and to reestablish civil engineering as a truly scientifically-oriented profession. The basic surveying course (or courses) must be well integrated with the requirements of the civil engineer and with his needs for the future.

There have been many accomplishments during the past 50 yr in the surveying and mapping field. Many will remember the extent of the requirements in surveying that were required of the civil engineer in the early part of this century. A recent review of a 1910 university catalog shows that at that time there were three required courses in surveying and two required courses in

Note.—Discussion open until July 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Surveying and Mapping Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. SU 1, February, 1960.

[a] Presented at the May 1959 ASCE Convention in Cleveland, Ohio.

[1] Associate Professor, School of Civil Engineering, Purdue University, Lafayette, Indiana.

the closely related subjects of railway location and construction; and there was still a large percentage of civil engineers being employed by the expanding railroad industry. There was no soil mechanics, no highway engineering, very few non-technical subjects (humanities and social sciences) in the curriculum and there were only a few less civil engineering graduates at that time than there are at present.

Through the years there has been a gradual de-emphasis, timewise, on surveying in the curriculum—giving way to the development and expansion of several other disciplines within civil engineering. To many it seems that this reduction has all taken place during the past 10 yr and yet it is a known fact that this has occurred over a great many years. The recent 1959 Brinker report on the status of education in surveying (delivered at the Los Angeles ASCE meeting in February, 1959) shows "the long-standing uneasiness about the stability of the credit situation in surveying." It shows that in 1937 the average required semester credits in our engineering colleges was 14.3 credits. This figure dropped to 11.3 credits in 1948, held rather firm through 1952, but has now dropped to an average of 7.7 credits.

This constantly decreasing emphasis on surveying certainly is not justified when one considers the great strides and developments that have taken place in this field in the past 10 to 20 yr. What then is the cause? Obviously much of the credit for this displacement must be attributed to the expansion of the other disciplines in civil engineering. However, much of the blame must be placed on the surveying teachers themselves for their complacency in the continual concentration of allotted time to the teaching of the manipulative skills and techniques of the surveyor through repetitive practice. Blame must also go, however, to a number of administrators who consistently assigned, and still do, the youngest and newest instructors to the teaching of surveying with the seeming attitude that "just anyone" can teach it. This practice also contributed to the retention of the elementary type course. It is no surprise, then, with the current curriculum revisions that criticism and suggested changes would be directed toward this area of Civil Engineering.

ASEE AND ASCE REPORTS

There are three rather recent reports which should be thoroughly studied by all people interested in surveying and mapping and civil engineering education.

A special study committee on the Evaluation of Engineering Education was appointed in 1952 by the American Society for Engineering Education. Three years later this committee made its monumental report which is having a great influence on engineering education throughout this country.[2]

It is pertinent to review the basic recommendation for engineering education as formulated by this important group.

> "Engineering Education must contribute to the development of men who can face new and difficult engineering situations with imagination and competence. Meeting much situations invariably involves both the professional and social responsibilities. The Committee considers that

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[2] Report on Evaluation of Engineering Education, The Journal of Engineering Education, ASEE, September 1955, Volume 46, No. 1, p. 25-60.

scientifically oriented engineering curricula are essential to achieve these ends and recommends the following means of implementation:

1. A strengthening of work in the basic sciences, including mathematics, chemistry, and physics.
2. The identification and inclusion of six engineering sciences, taught with full use of the basic sciences, as a common core of engineering.
3. An integrated study of engineering analysis, design and engineering systems for professional background, planned and carried out to stimulate creative and imaginative thinking, and making full use of the basic and engineering sciences.
4. The inclusion of elective subjects to develop the special talents of individual students, to serve the varied needs of society, and to provide flexibility of opportunity for gifted students.
5. A continuing, concentrated effort to strengthen and integrate work in the humanistic and social sciences into engineering programs.
6. An insistence upon the development of a high level of performance in the oral, written, and graphical communication of ideas.
7. The encouragement of experiments in all areas of engineering education.
8. The strengthening of graduate programs necessary to supply the needs of the profession, conducted in those institutions that can:
   a. provide a specially qualified faculty,
   b. attract students of superior ability, and
   c. furnish adequate financial and administrative support.
9. Positive steps to insure the maintenance of faculties with the intellectual capacity as well as the professional and scholarly attainments necessary to implement the preceding recommendations. These steps include:
   a. well-established recruitment, development, and evaluation procedures,
   b. favorable intellectural atmosphere, reasonable teaching loads, and adequate physical facilities, and
   c. salary scales based on the recognition that the required superior faculty can be secured only by competitive remuneration, since professional practice in industry and government is inherently attractive to the best minds in engineering.
10. The consideration of these recommendations at this time before the problems of educating greatly increased numbers of engineers become critical."

The time distribution for scientifically oriented engineering curricula have been suggested as a five part program.

About 25% on mathematics and the basic sciences of physics and chemistry

About 25% on the engineering sciences which include the mechanical phenomena of solids, liquids, and gases as well as electrical phenomena.

About 25% on a sequence of engineering analysis, design and engineering systems, including the necessary technological background.

About 20% on humanistic or social studies such as philosophy, history, English, literature, etc.

About 10% on options or electives so that the student can extend his interest toward his future professional activity and allows flexibility within the framework of a given curriculum.

It is somewhat ironical because the above adds to 105% and this seems to be one of our educational problems—to condense about 110% or 120% into only 100%. One of the solutions to this problem is to extend the curriculum to more than 4 yr.

Essentially there has been a shift of emphasis in our engineering education from the "how-to-do" to the "why".

Concerning engineering laboratories, the committee reports:

> "The laboratory is the means of teaching the experimental method. It should give the student the opportunity to observe phenomena and seek explanations, to test theories and note contradictions, to devise experiments which will yield essential data, and to interpret results. Therefore, laboratories should be used where and only where these aims are being sought. The value of a set number of stereotyped experiments is questionable. The development of a smaller number of appropriate experimental problems by the students themselves under effective guidance will have much greater educational value."

The further add, however, that

> "art of measurement—including analysis of accuracy, precision, and errors—and the appreciation of the degree of accuracy economically justified, together with some understanding of statistical methods, are essential elements of laboratory experience. . ."

The committee also states that

> "shop courses and all other courses emphasizing practical work that tend to displace engineering science in the curriculum should be scrutinized critically in the light of the instructional goals already discussed."

The second report which discloses recent developments in Civil Engineering education is the report of the ASCE task committee on Professional Education completed in 1958.[3] This study points out the short-comings of education in civil engineering through the results of an extensive opinion poll of the civil engineering profession.

The third recent study which should have careful attention is that of our special task committee on the Status of Surveying and Mapping.[4] As a result of this exhaustive and detailed study, and its subsequent approval (in February 1959) by the Board of Direction of ASCE, the four major categories of the surveying and mapping field—land surveying, engineering surveying, geodetic surveying, and cartographic surveying—have been reemphasized as a part of the civil engineering profession. Much credit must be given to this diligent committee for their valuable report. The classification chart listing the professional-level and the technician-level positions in the various categories is

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[3] Report of Task Committee on Professional Education, Civil Engineering, A.S.C.E., February, 1958, Volume 28, No. 2, p. 111-123.

[4] Report on Status of Surveying and Mapping in the United States, Surveying and Mapping, A.C.S.M., October-December 1958, Volume 18, No. 4, p. 432-437.

a most worthwhile product and will probably be referred to as a standard for many years.

Most engineers are now aware that the curricula of our engineering schools have been or are being revised drastically to an orientation based more strongly on the basic sciences of mathematics, physics, and chemistry, together with an extreme effort to concentrate on the basic principles and theoretical approach with a corresponding de-emphasis on the practical approach.

As suggested previously, surveying has generally been taught with too much emphasis on the practical approach and it is obvious, therefore, that criticism and suggested changes would be directed early toward this area.

FUTURE REQUIREMENTS

If the answers to the following questions were satisfactorily known, many of the educational problems would be solved. A study of these questions will indicate the problems confronting the teaching of surveying.

1. What are the various disciplines (or areas of opportunity) within the broad field of civil engineering?
2. Where do the civil engineering graduates go when they finish school—in what areas of civil engineering are they employed, or do they stay in civil engineering?
3. How many graduates actually know, when they are undergraduates, where they will eventually find their place of contribution to the profession? How many civil engineers knew, when they were a sophomore or junior in college that they would be employed in their present position?
4. How much does the average civil engineering graduate need to know about surveying and mapping?
5. Is the new civil engineering graduate expected to be a productive employee immediately upon employment by the industry or government? Where is the new graduate to receive his basic training and practical introduction to the field of his choice?
6. What are the objectives and goals of engineering education?

The schools of civil engineering must in the future concentrate on an effort to teach the professional-level surveying (as outlined in the status committee report) with just enough technician-level work so the student will have no great trouble in understanding the usefulness of the theoretical principles. What minimum instrumental contact is necessary to accomplish this professional-level instruction in a basic course for all civil engineering students—keeping in mind that many will eventually go into structural design, sanitary engineering, etc.?

Probably one of the best approaches to the subject in the future is that involving a rather rigorous analysis of errors and accuracy in surveying measurement with a limited amount of instrumental work to illustrate the effect of the control of systematic errors. The basic course should also include a thorough grounding in control systems and datums for engineering surveys including directional control through engineering astronomy. Also the student should be exposed to the effects which the size and shape of the earth has on measurements and to the problems involved in mapping an area by both ground and photogrammetric methods, in subdividing and describing an area, and in locating and positioning man-made structures.

Beyond the basic course in surveying, as it is finally evolved as a core curriculum course, additional courses in the form of technical electives must be available both at the undergraduate level and, at larger institutions, at the graduate level to provide for further professional-level surveying, that is, in land surveying, geodetic surveying, and cartographic surveying. The justification for offering these courses in the smaller engineering schools with limited student enrollment is a definite problem.

Perhaps, if almost all of the practical aspects are eliminated from the core curriculum course, it may be desirable to offer an applied surveying elective for those students who still desire a practical project type course in order to gain an insight into the planning and execution of various types of surveys. This is generally the type of course that a good summer surveying camp involves. Many students and some uninformed faculty members think that this type of training can come from summer jobs; but only in a very small percentage of summer jobs can a diversified surveying experience be obtained. Too frequently the student is retained as a rodman or a chainman throughout his period of employment. Many students cannot see, (from their summer work) the professional-level surveying—they cannot see the woods because of the trees! As a passing thought—with the increased pressure to delete the practical aspects in favor of the more theoretical approach—it is going to become increasingly more difficult to justify the existence of a summer surveying camp program.

One might logically be concerned, then, about the training of the large number of technician-level surveyors which are required in the actual execution of the many surveying projects. This is an area which must be greatly extended in the near future through the expanded offerings of the 2-yr technical institutes. A big problem exists in this area because it has been hard to encourage good students to enroll in this essential type of program. We must double or triple our efforts to make these technician-level programs more satisfying and rewarding as a career.

J. D. Ryder of Michigan State University in a talk on professional manpower problems before the American Congress on Surveying and Mapping in Washington, D.C., March 12, 1959, strongly urged the civil engineering field to become more fully aware of the possibilities inherent in the employment of graduates of the 2-yr technical institutes.

In connection with the problem of future government employees in surveying and mapping, the fact should be emphasized that the civil engineering graduate of the future will be well equipped at commencement time with a thorough knowledge of the basic sciences and theoretical principles upon which he can build toward intellectual maturity at a higher level than was previously attained. The employers of the future must realize, however, that the graduating civil engineer will not be a productive individual directly upon graduation, but after an in-service training period, the new civil engineer should yield a much greater return in the long run. As Br. Austin Barry of Manhattan College points out, future civil engineering graduates may have very little dexterity in the use of the chain, but on the other hand, they may be able to show their employer how to throw the chain away in favor of some electronic measuring device.

Above all, employers must make proper use of professional-level personnel in their organizations. Too many instances of the improper use and hoarding of professionally trained engineers have been reported in the past. Many positions now held by engineers can be satisfactorily handled by a graduate of a technical institute.

The adopted status committee's report states "that fully accredited civil engineering curricula should include adequate instruction in surveying by qualified personnel." Considerable study will be necessary to determine what constitutes "adequate instruction." A well-qualified teacher of surveying should have had a considerable amount of varied surveying experience and should have a thorough knowledge of errors and accuracy, control systems and datums, geodetic relationships, advantages and uses of the state plane coordinate systems, and an understanding of the problems of mapping an area by both ground and photogrammetric methods, problems in land surveying, and problems in the generation of dimensions, i.e., construction layout.

Obviously this background requires a teacher with more than only a passing interest in the subject material. Modern developments and new applications are coming fast and it is difficult for one who specializes in teaching surveying and mapping to keep up—let alone an instructor who must also teach structures, hydraulics, etc., concurrently.

IS CIVIL ENGINEERING DIFFERENT?

Recently an extensive study was made of the careers and opinions (about engineering education) of the Purdue University graduate. Some 3,800 engineering alumni of Purdue from the classes of 1911 to 1956, inclusive, responded to this questionnaire. Regarding the major field of employment, 73.4% of the engineering graduates were employed by private industry, 9.4% were employed by the government, (7% of which was in federal service), 5.0% in private engineering practice, 4.4% were self-employed in non-engineering work, 4.1% in educational institutions, and the remaining 3.7% were retired, housewives, or employed in some other category. This was the distribution for all engineers.

The distribution, however, for the civil engineers as a separate group was quite different. Only 51% of the civils were employed in private industry, 21% of the civils were in government and 15% were in private engineering practice.

This is important because it would seem to show that the civil engineer is of a breed which is a little different from the mechanical, chemical, or electrical engineer. In no other branch of engineering, is it as possible as it is in civil engineering to gain some experience after graduation, then proceed to hang out one's "shingle" to practice private engineering. What type of undergraduate education should this type of individual private practitioner receive? Perhaps, civil engineering should be a little more cautious in curriculum revisions and not quite so anxious to follow too closely the curricula changes in the other branches of engineering.

A number of educators and engineers are considerably concerned that civil engineering can go too far away from an applied science. John B. Wilbur has stated[5]

> "The pendulum of engineering education which, some years ago, had doubtless swung too far in the direction of engineering practice, seems now in danger of too much rebound into the realm of science. . . . I shall use my influence to minimize the overswing. We would not think for a

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[5] An Excursion Through Exactitudes, Journal of Engineering Education, December, 1957, Volume 48, No. 3, p. 182-187.

moment of dropping the human values from engineering education; and we should guard the judgment values with an equal zeal."

Russell C. Brinker, Professor and Head of Civil Engineering at Texas Western College, believes[6]

TABLE 1.—OPINIONS ABOUT THE GOALS OF GENERAL EDUCATION AT THE COLLEGE LEVEL

Goal	Relative Order of Importance
To express one's thoughts effectively	1
To acquire and use the skills and habits involved in critical and constructive thinking	2
To learn to get along with people	3
To understand the ideas of others	4
To develop knowledge and understanding in making possible a more effective choice of one's life work	5
To become proficient in one's chosen field of work	6
To attain emotional and social judgement	7
To develop a code of behavior based on democratic and ethical principles	8
To experience a realistic sampling of one's chosen vocation	9
To move smoothly from adolescent dependence to adult independence	10
To habitually apply scientific thought to the discovery of facts	11
To acquire knowledge and attitudes basic to a satisfying family life	12
To master certain techniques applicable to one's vocational field of special interest	13
To develop a broad general outlook and familiarity with a variety of subjects	14
To maintain and improve one's health	15
To know the major developments in a field of vocational or special interest	16
To bring up to date one's knowledge in a special field of interest or a vocational field	17
To understand one's physical and social environment	18
To acquire specific information and techniques in preparation for further study in a particular field	19
To acquire a degree of experience in a special field	20
To recognize the fact of world inter-dependence	21
To develop the ability to do independent research	22
To master a classification of knowledge in a field	23
To develop certain manual skills	24
To understand other cultures and people	25
To understand and enjoy literature, art, and music	26

". . . that changes being made in civil engineering curricula desirable, necessary, and could be carried still further for the top 15% of 'engineering' students who will assume the highest levels of analysis,

6 Status of Education in Surveying, paper presented at February, 1959 ASCE, Los Angeles Convention.

design, and research. The overwhelming majority of engineering students, however, are likely to benefit more from a program having some practical content. . . ."

TABLE 2.—OPINIONS ABOUT THE GOALS OF ENGINEERING EDUCATION

Goal	Relative Order of Importance
Ability to think straight in the application of fundamental principles to new problems	1
Ability to organize thought logically	2
Mastery of fundamental scientific principles	3
Ability to present significant results in a study clearly in oral and written forms	4
Ability to express thoughts lucidly and convinceingly in oral and written English	5
Grasp the meaning of physical and mathematical laws	6
A foundation for engineering judgment	7
Thorough understanding of the engr. method, and elementary competence in the application of the engineering method	8
Ability to read with discrimination and purpose	9
Resourcefulness and originality in devising means to an end	10
Command of basic knowledge of branches of engr. (EE, ME, etc.)	11
Understanding the element of cost in engr. and ability to deal with the element of cost just as competently as with technological factors	12
Reasonable skill in choosing the type of approach in light of the time available for solution	13
Comprehending interacting elements in situations which are to be analyzed	14
Development of moral, ethical, and social concepts essential to a career and consistent with the public welfare, to a satisfying personal philosophy, and to a sound professional attitude	15
Reasonable skill in making approximations	16
Knowledge of how physical laws evolve and the limitations of their use	17
Stimulation of a continued interest in further professional development	18
Knowledge of materials, machines, and structures	19
Ability to recognize, make a critical analysis of, and arrive at an intelligent opinion about a problem involving social and economic elements	20
Understanding the evolution of the social organization in which we live and the influence of science and engineering on its development	21
Attainment of an interest and pleasure in social-humanistic pursuits and, thus, an inspiration to continue study in this area	22
Acquaintance with some of the masterpieces of literature, and an understanding of literature's setting and influence on civilization	23

Two of the results of the above-mentioned Purdue opinion poll were the summaries of opinions concerning the relative importance of the goals of general education at the college level (Table 1) and opinions concerning the relative importance of the goals of engineering education (Table 2).

EXPERIMENTATION

Certainly one of the necessities of the future in surveying and mapping education, as well as in all of the civil engineering education, is the willingness to experiment with different methods of presentation—the educators must not become attached to a fixed pattern with a tendency towards stagnation.

The experiments of Ohio State University and George Washington University with separate curricula in surveying and mapping are to be commended. The recently proposed program of teaching surveying to high school students in Miami, Florida, is to be encouraged and tested. Educators should not shy away from these experiments.

Theodore Roosevelt once said

> "It is not the critic who counts; not the man who points out how the strong man stumbled, or where the doer of deeds could have done them better.
>
> "The credit belongs to the man who is actually in the arena; whose face is marred by dust, sweat, and blood; who strives valiantly; who errs and comes up short time and again; who knows the great enthusiasms, the great devotions, and spends himself in a worthy cause; and who at the best knows in the end the triumphs of high achievement; and who at the worst, if he fails, at least fails while daring greatly, so that his place shall never be with those cold and timid souls who know neither defeat nor victory."

CONCLUSIONS

The current curriculum revisions which are taking place in the schools of civil engineering in the United States are essentially designed to reaffirm and to reestablish civil engineering as a truly scientifically oriented profession. In the future, less and less time will be devoted to the art of engineering on our college campuses and more and more time on the basic theoretical principles of science upon which the new engineering graduate can build toward a higher level of intellectual achievement. The employers of the future must realize that the newly graduated civil engineer will not be a productive individual directly upon graduation but, after participating in an in-service training program, this civil engineer should yield a greater contribution.

With the thresholds of science being extended at a rapid rate, it is imperative that the civil engineer be better equipped to successfully cope with the complex problems of the future. This pertains to the field of surveying and mapping just as it does to all the other fields of civil engineering. The survey engineer of the future will, of necessity, become more involved in electronics, photogrammetry, geodesy, and astronomy—as the general case, rather than the exception. Accompanying this increasing demand for more adequately and broadly trained professional civil engineers, is the necessity of expanding and utilizing the graduates of the 2-yr technical institute programs to fill the need for men trained to handle the operational details.

Journal of the
SURVEYING AND MAPPING DIVISION
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CIVIL ENGINEERING CURRICULUM FOR FEDERAL SURVEYING AND MAPPING

By G. C. Tewinkel,[1] F. ASCE

SYNOPSIS

The federal government has historically employed civil engineers in those branches concerned with surveying and mapping. In recent years, however, the recruitment efforts have fallen short of the necessary levels. The author discusses the reasons for this turn of events and analyzes curriculum requirements.

MAPPING IS CIVIL ENGINEERING

The United States Geological Survey (USGS) has historically employed civil engineering graduates as the core of its mapping personnel. The survey is being successful in maintaining this principle in spite of the short supply of engineering graduates.

The USCGS is guided by a corps of about 170 commissioned officers of whom about 95% are civil engineer graduates. Each year about 40 new graduates are recruited for this service. (Of these, only about 4 remain after their Selective Service requirements are met.) Although many of the older civilian supervisory personnel have civil engineering degrees, the USCGS is having difficulty in filling their vacancies with qualified engineers.

The other mapping agencies also have preferred civil engineers for several reasons. The civil engineer, in the past at least, had training in surveying:

Note.—Discussion open until July 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Surveying and Mapping Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. SU 1, February, 1960.

1 Chief, Research Branch, Photogrammetry Division, Coast and Geodetic Survey, United States Department of Commerce, Washington, D. C.

traverse, triangulation, levels, topography, computations. He seemed to prefer an outdoor profession. He was able to adapt himself to a wide variety of technical tasks including hydrographic surveying, instrument design, and even the mathematics of geodesy. He was usually a good manager whether it was as a chief of party, captain of a survey ship, bureau personnel chief or bureau director. In this sense, a civil engineer has been regarded as a super jack-of-all trades. Today his professional status has acquired even greater significance because of recent enactments of state laws requiring licensing or registration of all engineers who are in responsible positions with regard to public works, including public land surveys.

A few years ago it seemed as though the surveying and mapping discipline was about to be weaned from civil engineering. But today it is different. ASCE has emphatically announced that surveying and mapping is a part of civil engineering. And the United States Civil Service Commission has recognized that civil engineers are necessary in significant positions in the surveying and mapping agencies.

The civil engineering curriculum attracts to its student body young minds which function like civil engineer's minds—those engineer-like "personalities" mentioned by Elmer K. Timby.[2] Thereafter the curriculum rigorously trains them in the solutions of a wide variety of theoretical and practical scientific problems. This type of mind so trained has great value in surveying and mapping as well as in industry. It may be significant that the engineering student gets almost 5 yr of college training compressed into 4 yr of time as compared to some of the other disciplines. Thus industry gets more for its money—gets hard-working masters for the price of bachelors. (Perhaps this partly explains why engineer graduates are paid well.)

Consequently, it is rather easy to understand why the engineer graduate has become so popular among all sorts of industries entirely different from surveying and mapping. In fact his services are in such great demand today that he has almost abandoned the surveying and mapping field for "greener pastures" —or is it that government surveyors have failed to maintain green-looking pastures?

FEDERAL HIRING IS OFF

A relatively small number of civil engineers have been hired during the past few years for civilian career employment in the surveying and mapping field by the USCGS, the Geological Survey, the Navy Hydrographic Office, the Army Map Service, or the Aeronautical Chart and Information Center. Notable exceptions are the recruitment programs of the Geological Survey and the USCGS and the fact that some agencies have somewhat of a non-engineering, small-scale map compilation function to perform.

The reasons these graduates were not hired are (a) they did not apply for employment, and (b) the government, with the exceptions noted above, has no forceful recruitment program to compete with that of industry. Naturally, other contributory reasons exist. Ordinarily, starting salaries for government service are still lower than those of industry even though the government has recently taken significant remedial measures. Also, some persons just don't like the idea of working for the government—to some it does not seem to be in good

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[2] Civil Engineering magazine, March, 1959.

taste. Federal employment seems to have the reputation of offering few opportunities for advancement or achievement, which is not true. Surveying may seem to lack the glamour connected with some industrial engineering careers. Lower classmen who might have specialized in surveying may not have been certain that the field was sufficiently large to absorb them. Industry has been able to promise jobs for all graduates, whereas government organizations are ordinarily unable to make such statements as "we have an annual requirement for N civil engineering graduates".

The historical glamour of the booted surveyor in our western wilds seems to have faded. Although glamour is an emotional entity not necessarily supported by fact, the idea is difficult to counteract in the mind of the youthful engineering student. Today's graduate may not be the outdoor type of those of former generations. This may be understandable if one considers that the high school graduate may be attracted to engineering not so much because of a promise of an outdoor life but rather because of a suggestion of a home not far from his engineering laboratory and office. The younger marriage age during recent years also has had an effect—as husbands they do not want to be away from home on distant survey projects, whereas formerly a larger proportion remained single until after they had completed a large part of their field service.

WHAT ABOUT CARTOGRAPHY?

In the government mapping agencies are found specialized skills in cartography along side of those in engineering. ASCE has spelled out the distinction between the two fields and the distinction is recognized by the Civil Service Commission. In one sense, cartography is associated with geography; in another it is associated with the compilation and publication of charts and drawings containing basic engineering information obtained by engineers and engineer technicians. So one sees that the two fields are interwoven; but they are nevertheless separable.

At least three universities in this country offer academic degrees in cartography: George Washington University, Ohio State University, and University of Washington (Seattle). None of these schools are in the engineering colleges.

Government mapping agencies use a relatively large number of employees with such training. A large part of the federal mapping work has to do with charts and maps of fairly small scale, which fits in well with cartographic specialization. Some of the cartographic courses taught in these schools venture close to the engineering field by requiring study in mathematics, surveying, photogrammetry, geodesy, map projections, planning, etc. But the writer believes the real distinction lies in whether or not the individual graduate can qualify as a registered professional engineer in a given state. If he is a graduate civil engineer with sufficient beginning experience, he can qualify readily; if not, to qualify is most difficult.

How and where specialized training in surveying and mapping are to be obtained is difficult to answer. Consider such subjects as geodesy, theory of errors, least-squares solutions, triangulation-traverse-trilateration computations, theory and adjustment of photogrammetric surveys, theory and application of map projections, geophysics, etc. These subjects receive very small consideration in the United States schools, and perhaps have little place in modern undergraduate engineering study. But then where can a specialist get this sort of information? A partial answer is that he can get this training in his own organization through on-the-job training courses. The profession is generally aware that these specialties are taught in several European technical schools, and happily some of these graduates have migrated to the United States to fill voids in our own system and to add fresh thought to these academic topics. But it should perhaps be borne in mind that if such courses are not taught frequently, the art, personnel, and necessary enthusiasm for teaching them may eventually be lost.

INFLUENCES ON FUTURE STUDENT ENROLLMENTS

Three factors have already been mentioned which may have a favorable effect on student enrollment in surveying and mapping curricula: (a) the acceptance in February, 1958 by ASCE of the recommendations of the Task Committee on Status of Surveying and Mapping recognizing certain activities as being a part of civil engineering; (b) the new standards of the Civil Service Commission recognizing civil engineering training as needed in many positions in surveying and mapping agencies; and (c) the continual trend of most of the states in requiring professional engineering registration in order to practice within the state, including land surveyor registration.

A factor which may have a significant effect on the number of students attracted to surveying and mapping is the relatively recent organization of the two national societies in this field: the American Society of Photogrammetry and the American Congress on Surveying and Mapping. (As a third one, the land surveyors are beginning to consider national organization.) These organizations are young and, as yet, not very strong, but they will surely grow and mature into influential societies. Their influence may be subtle: they will serve to focus attention onto the surveying and mapping science which formerly had no organization to give voice to its needs, and which had been overshadowed by other phases of professional engineering.

Another force which may make itself felt more strongly in the future is the profession of land surveyors. First, ASCE recognizes him now as being a professional engineer. Secondly, most of the states have registration statutes which are readily complied with by civil engineering graduates. Thirdly, the population growth of the nation is increasing the relative values of land, creating greater interest in land surveying, and increasing the importance of the surveyor to protect and subdivide properties. Fourthly, it is generally recognized that the land registry and subdivision system of the United States is not a very modern one compared to contemporary European systems. When this begins to be corrected, even greater attention will be turned toward the land surveyor. In turn, universities will arise to the occasion in supplying the needed civil engineers.

It is not expected that all universities can offer full specialization in surveying and mapping even to graduate students—the demand is just not sufficient. But there probably is a need for a half dozen of them. The pity of it is that there are not a half dozen of these institutions today, and even those few do not have a sufficient number of surveying and mapping students to warrant scheduling all of the courses. It is hoped that all the foregoing influences will eventually flood these schools with students.

THE CHANGING PERSONNEL STATUS

It is not difficult to recognize that certain changes which have occurred and are occurring in the relationship of civil engineering to governmental surveying and mapping are due, in part at least, to forces which relate to the changing times. Wars and peace, and their resultant birth-rate disturbances are important factors as they relate to the changing rates of available engineering graduates. The cold war has undoubtedly had a great effect; the tremendous scientific military development has placed an unprecedented demand on the supply of engineers and scientists of all categories. A constantly high business level for nearly 15 yr following World War II without a serious depression is a forceful factor. A contrary force existed in the 1930's which sent a flood of highly qualified engineers into federal service where they remained for the balance of their careers. The result today is a high tide in the demand for civil engineers which will not be stemmed.

Today is found a strange age strata of engineers in some, if not all, of the government services. An abundance of engineers exists about the 45-yr age level; a dirth exists among the younger engineers, the explanation being in the foregoing factors.

As a result of the difficulty in obtaining a sufficient number of engineers in the lower echelons, at least three of the agencies have taken definite training steps to remedy the deficiency. No two programs are exactly alike, but all of them are similar in the sense that they encourage employees to complete their university educations so that they can qualify as professional civil engineers (and also other disciplines). All of the programs serve to retain nearly all training within the university. The programs enable worthy students and tested employees to obtain bona fide degrees. The financial aids to the students vary from simple encouragement, summer or part-time employment, to tuition payment, as well as cooperative study-work arrangements and night school attendance. Further, some agencies have invited the local universities to hold classes within the government facilities both during and after working hours. In addition, some courses are taught by qualified agency personnel during work hours without academic status; in a few instances the university awards academic credit if the necessary conditions are met.

Naturally, training programs are expensive to the government, but in this instance they present the only solution to the problem of getting qualified professional civil engineers into pertinent surveying and mapping positions—the only way of filling this engineer void below the age of forty.

CONCLUSIONS

Essentially all of the training and education needed in the surveying and mapping field is available within American universities. Too few students are taking advantage of this availability to meet the recruitment programs of the government agencies. It is expected in the future that, in view of the many factors cited, a large number of students will decide during their college to make their careers in surveying and mapping. In the meantime, the agencies themselves have instituted training programs to help fill current needs. And finally, and of paramount significance, it is more important that the graduate be an engineer rather than whether or not he has studied certain specialized courses.

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TOPOGRAPHIC MAPPING IN ALASKA[a,b]

By Reynold E. Isto,[1] M. ASCE

SYNOPSIS

Alaska has presented many challenges to mapmakers in the form of terrain, weather, and logistics problems. At the same time it has been an opportune proving ground for the development of new and enterprising techniques. An outstanding example of Alaskan mapping projects was that recently completed in the Brooks Range Area. The Alaskan mapping accomplished to date has been a joint effort of the Geological Survey, Coast and Geodetic Survey, and Department of Defense.

INTRODUCTION

The task of accomplishing the topographic mapping of Alaska has presented a formidable challenge to mapmakers. Yet, in spite of severe difficulties, Alaska enters upon statehood with its terrain more completely mapped than any of the other States at the time of admission.

Note.—Discussion open until July 1, 1960. To extend the closing date one month, a written request must be filed with the Executive Secretary, ASCE. This paper is part of the copyrighted Journal of the Surveying and Mapping Division, Proceedings of the American Society of Civil Engineers, Vol. 86, No. SU 1, February, 1960.

[a] Presented at the October 1959 ASCE Convention in Washington, D. C.

[b] Publication authorized by Director, U. S. Geological Survey.

[1] Resident District Engineer, Topographic Division, U. S. Geological Survey, Fairbanks, Alaska.

GENERAL PROBLEMS

Alaska, because of its geographic location, has many mapping problems which are not encountered in States to the south. For example, over one-half to three-fourths of the area is underlain by either continuous or sporadic permafrost (perennially frozen ground which has remained frozen for 2 yr or more).

On the arctic slope, frost mounds called pingos, have been found useful as observation points for triangulation, because of their height above surrounding terrain. Aufeis (a German term for icing or flood ice) on the river channels has created puzzles for the photogrammetrists. Alaska has hundreds of glaciers; each of the two largest has an area roughly equivalent to that of Rhode Island. These glaciers have created interpretation problems for the map compilers. The problem of photointerpretation of Alaskan woodland types has also given rise to many controversies.

Alaska is an area of adverse and widely different weather conditions. The coldest recorded temperature in North America (-81° F.) occurred at Snag, Yukon Territory, just 20 miles from the Alaska-Canada border. Surprisingly, this location is not near the northernmost point in Alaska but is actually 550 miles south of Point Barrow. Conversely, a temperature of 100° F., the highest ever recorded in Alaska, occurred at Fort Yukon which is north of the Arctic Circle and 825 miles north of the southernmost point in Alaska. Annual rainfall varies from a maximum of 269 in. at Little Port Walter in southeastern Alaska, to less than 5 in. on the arctic slopes of the Brooks Range. Inclement weather conditions in the coastal areas have at times prevented field parties from working for as long as 4 consecutive weeks, but early summer weather in the interior can be so good that, with the continual daylight, engineering and aircraft crews often find themselves exhausted and hoping for bad weather.

HISTORICAL BACKGROUND

The historical background of mapping activities in Alaska has been summarized by George D. Whitmore[2] as follows:

"During the long Russian occupation of Alaska, the Russians and many others made exploratory surveys of Alaska's coastline. After purchase by the U. S. over 90 years ago, military exploratory parties began the penetration of Interior Alaska. Organized mapping, including that of the Geological Survey, began shortly before the turn of the century, at about the time of the Klondike gold rush.

"Before World War II, Alaska had been about half covered with a non-uniform series of maps, the greater part accomplished by plane-table surveys in the field, but with important support coming from pioneering efforts in terrestrial and aerial photogrammetry. Most of the maps were reconnaissance in nature, 1:250,000 scale with 200-foot contour intervals, prepared by joint geologic-topographic parties of the Geological Survey. This kind of combined operation, although successful at the time, was subsequently abandoned in favor of separate and more specialized modern operations. In the early work, a number of areas were mapped at 1:62,500 scale, in regions of mining interest, including the Alaska Rail-

[2] Ninth Alaska Science Conference, September, 1958.

road Belt, the Yukon-Tanana country, portions of southeastern Alaska and Seward Peninsula.

"Other agencies also were active in Alaska mapping prior to World War II. The International Boundary Commission mapped the Alaska-Canada border in southeastern Alaska and north to the Arctic Ocean. The 29th Engineer Topographic Battalion mapped a large part of the Aleutian Islands and about 7,000 square miles in the vicinity of Anchorage. The Forest Service compiled planimetric maps of the Tongass and Chugach National Forests and the Coast and Geodetic Survey mapped scattered coastal areas.

"In recent years, the 30th Engineer Topographic Battalion has performed extensive control surveys in western Alaska and on the Alaska Peninsula, for compilation by the Army Map Service, and has undertaken additional control surveys in central and western Alaska, for compilation by the Geological Survey. The complete mapping of the Aleutian Island chain is being accomplished by the Coast and Geodetic Survey.

"The Geological Survey has welcomed the cooperation of other Federal mapping agencies in accomplishing the Alaska mapping. Coordination of all Federal efforts has been fundamental in the Alaska mapping program since the beginning. By the joint efforts of all concerned, the program has proceeded rapidly, and many important deadlines have been met.

"The Geological Survey has prepared a series of Alaska base maps, each of which covers the entire area of Alaska, at varying scales. They are designated by letters as follows:

Map A	1:5,000,000
Map B	1:1,584,000 (available with contours)
Map C	1:12,000,000
Map E	1:2,500,000 (available with shaded relief)"

DEVELOPMENT OF SPECIAL TECHNIQUES

The challenge of Alaska's sheer size and relative remoteness which confronted the early explorers has been met by the introduction and development of modern mapping techniques which are certain to find application in other unmapped areas of comparable difficulty. The chief innovation of recent years has been the use of air transportation, particularly helicopters, as described by Gerald FitzGerald.[3]

The first photography used in mapping Alaska was the terrestrial photography used by the Canadian Government and the United States Coast and Geodetic Survey (USCGS) on the International Boundary Surveys in southeastern Alaska in 1893. A panoramic camera was developed by the United States Geological Survey (USGS) in 1905 and used to great advantage on topographic surveys in Alaska. The panoramic photographs were later used in a panoramic photoalidade developed between 1910 and 1916. The first extensive aerial photography in Alaska was obtained by the United States Navy in 1926 and 1929. This photography was used by the USGS to make planimetric maps covering approximately 20,000 sq miles in southeastern Alaska.

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3 "Helicopter Revolutionizes Topographic Mapping of Remote Areas," in the December 1954, issue of Civil Engineering.

In 1941 the Alaskan Branch of the USGS was asked by the Army Air Corps to develop a rapid reconnaissance type of mapping for use in compiling aeronautical charts. This request resulted in the development of trimetrogon photography and the improvement of the photoalidade. Using this instrumentation, a program of compiling small-scale reconnaissance maps was carried on for a number of years and eventually resulted in complete coverage of Alaska with aeronautical charts and 1:250,000-scale maps.

THE MODERN PROGRAM

The years 1948-1949 marked a turning point in Alaskan mapping activity. The USGS's mapping program was doubled in 1948 and tripled in 1949, with increased demand for map coverage from the Department of Defense and civilian map users.

Since 1948 virtually all Alaskan topographic mapping has been carried out by compilation from aerial photographs. Compilation has been accomplished mostly with multiplex and ER-55 stereoscopic plotting instruments.

Field work in Alaska has consisted of two distinct phases: first, the procurement of sufficient vertical and horizontal geodetic control to satisfy photogrammetric needs; and second, the classification and photoidentification of cultural features.

CONTROL

Before and during the implementation of the modern mapping program, the USCGS established first-order triangulation arcs along the coastal perimeter of Alaska and in the interior. This agency also established high-order bench marks along most of the roads and railroads. In addition, elevations were established by the vertical-angle method on most of the triangulation stations. These surveys are used as basic control for topographic mapping.

The first step in the USGS procedure of developing control for mapping is to establish supplemental triangulation of third-order accuracy with sufficient density to fulfill the requirements for photogrammetric horizontal control. In most cases this has been accomplished with optical-reading theodolites which measure horizontal and vertical angles to seconds of arc. Elevations have been carried forward to all triangulation stations by the reciprocal-vertical-angle method.

The next step is to obtain adequate elevations to satisfy the photogrammetric compiler's needs. The elevations of the existing bench marks and triangulation stations meet part of the requirement. In coastal areas, the numerous bays, canals, inlets, and other salt-water features furnish many sea level elevations. Some vertical control is obtained by stadia traverse from points of known elevation but most of the vertical control is established by precise altimeter surveys using the one-base, two-base or leap-frog methods, with helicopters for transportation. Experience indicates that altimeter surveys conducted with care produce results that are accurate within about 10 feet.

Another common technique for establishing supplemental vertical control is the photo-trig method. In this method vertical angles are measured from triangulation stations, or other points of known elevation, to photo-identifiable and visible points such as mountain tops. Distances are determined by photogrammetric methods and differences in elevation are then computed by trigo-

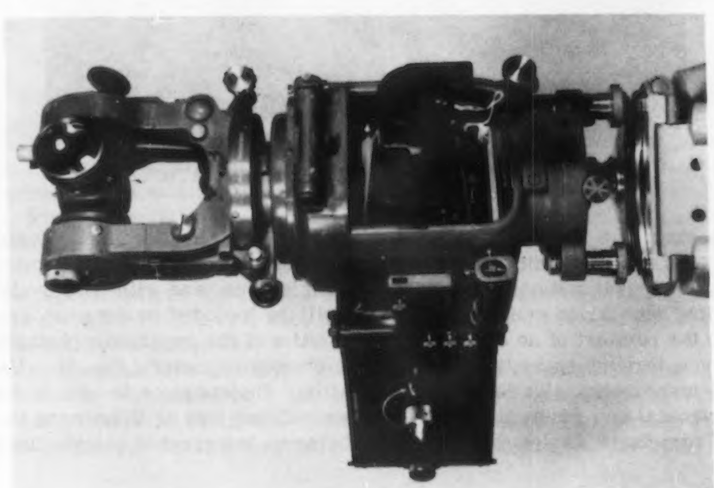

FIG. 1.—PHOTOTHEODOLITE INSTRUMENT.

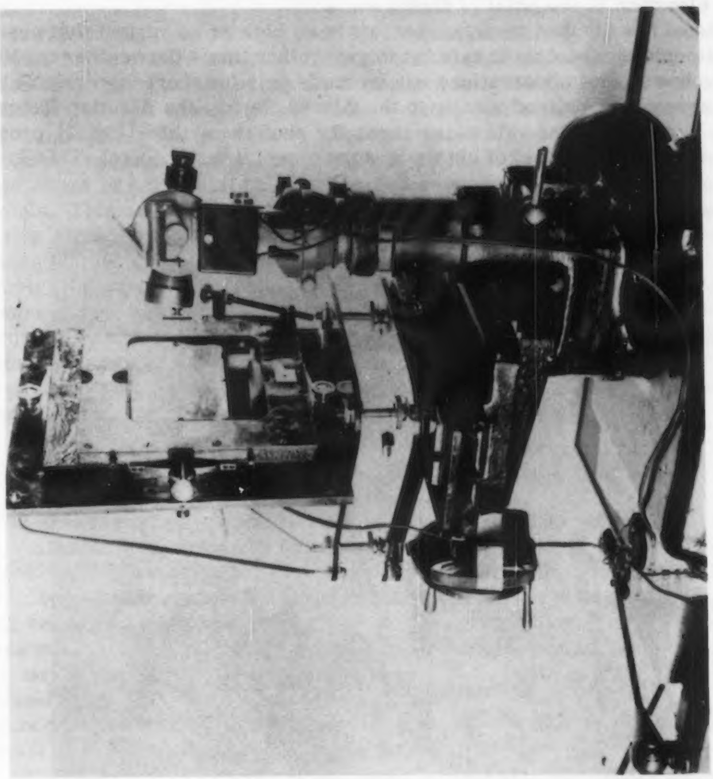

FIG. 2.—PHOTOGONIOMETER INSTRUMENT.

nometry. Occasionally, vertical angles are read from the photo-point of unknown elevation to the point of known elevation.

Some of the Alaskan mountain terrain is so high or so rugged that present-day helicopters cannot make safe landings; at other times the weather conditions are so adverse that observations can be made only for short intervals of time. These areas are confined mostly to the Alaska Range, the Aleutian Range and the St. Elias Mountains. In these areas the phototheodolite (Fig. 1) provides the most effective method of obtaining supplemental map control. This instru-

FIG. 3.—TELLUROMETER UNIT IN OPERATION.

ment consists of a theodolite and terrestrial camera mounted on a common vertical axis. Panoramic photographs are taken from existing triangulation stations. Vertical and horizontal azimuth angles are read with the theodolite to a limited number of critical points that will be included in the photograph. Later, in the comfort of an office, a glass positive of the panoramic photograph is observed through an instrument called a photogoniometer (Fig. 2). Using the field-measured angles to orient the positive, the operator is able to measure horizontal and vertical angles for intersections just as if he were in the field. From these angles and distances between intersected points, the positions and elevations of horizontal and vertical control points may be computed.

A variation of this method, used by field parties since 1949. employs a theodolite and a Polaroid Land camera. Panoramic photographs of the horizon are first taken at the occupied triangulation station. Then, vertical and azimuth angles are read to points such as sharp peaks that are identifiable on the photographs. Later, nearby triangulation stations are occupied and the procedure repeated, reading on the same points. From these observations, computations of horizontal positions and elevations can be made for the sighted points. This and the phototheodolite method have been used extensively for mapping mountainous terrain. Such techniques proved particularly useful in mapping Mt. McKinley which rises over 20,000 ft above sea level.

During the summer of 1958 the Tellurometer, (Fig. 3) an electronic distance-measuring device, was used for obtaining much of the horizontal control. This instrument has been used for field parties in the other States for the last two seasons with much success. In Alaska, it was especially useful in traversing across large glaciers and along the beaches where it would have been difficult to undertake triangulation arcs.

LOGISTICS

One of the most important aspects of Alaskan mapping operations is logistics. Alaskan road networks are sparse, and the use of trucks has been very limited. Early mapping parties traveled by pack train, dog team, and river boat. The airplane was first used for transportation of mapping parties in 1928; recently the use of aircraft by field parties has become increasingly extensive. To make the most efficient use of helicopters (Fig. 4), light fixed-wing airplanes (Fig. 5) are used to support base camps. These planes are equipped with float, wheel, ski-wheel or tandem-wheel landing gear, in accordance with the particular need. To move camps and bulk supplies larger aircraft of 3-1/2-ton to 5-1/2-ton (Fig. 6) capacity are often used. Alaska has approximately four hundred airfields but in some parts, such as on the arctic slope, there is a scarcity of airfields that can accommodate large aircraft. In these areas, the bulk supplies have been transported during the winter and early spring, using frozen lakes for landing fields.

BROOKS RANGE PROJECT

In December 1954, the USGS started planning the Brooks Range mapping project, comprising about 120,000 sq miles, equivalent in area to New Mexico. The project area extended 500 miles westward from the Alaska-Canada boundary, with the south boundary just south of the Arctic Circle, and the north boundary just south of the Arctic coast. The terrain ranges in elevation from near sea level to about 9,000 ft (Figs. 7, 8 and 9).

The specifications called for an accurate photogrammetrically compiled map at a scale of 1:250,000, with a general contour interval of 200 ft and 100-ft contours in the flat areas.

To accomplish this immense mapping assignment most expeditiously, the plan called for the use of twin low-oblique transverse aerial photography taken at an elevation of 30,000 ft above terrain. Early in 1955 proposals were submitted by private contractors for the aerial photography. In spite of many

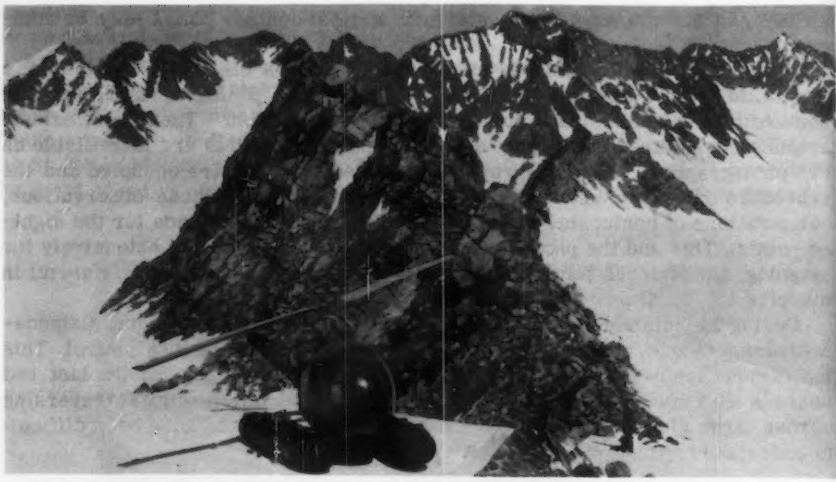

FIG. 4.—THREE-PLACE HELICOPTER NEAR MT. McKINLEY.

FIG. 5.—SMALL AIRPLANE USED TO SUPPORT SURVEY PARTIES. THIS ONE IS EQUIPPED WITH TANDEM WHEEL LANDING GEAR.

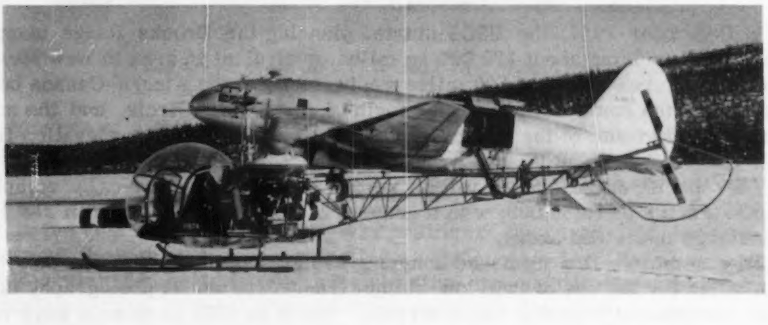

FIG. 6.—HELICOPTER AND A 5 1/2-TON TRANSPORT AIRPLANE.

FIG. 7.—TYPICAL MOUNTAIN TERRAIN IN THE WESTERN PART OF THE BROOKS RANGE.

FIG. 8.—MOUNTAIN TERRAIN IN THE ROMANZOF MOUNTAIN AT THE EASTERN END OF THE BROOKS RANGE.

difficulties, the contractor obtained approximately 100,000 sq miles of suitable aerial photography in the eight days of favorable photographic weather that occurred in 1955. The remainder of the aerial photography was completed the following summer.

A complete evaluation was made of the existing control which consisted of triangulation around the perimeter and one north-south arc of triangulation through the center of the project, established by the USCGS in 1955. One additional north-south triangulation arc was planned so that the interior horizontal control could more readily be extended by the stereotemplet method (Fig. 10). This arc was only partially completed the following field season, but it helped the stereotemplet solution, nevertheless.

To provide the basis for establishing vertical photogrammetric control, fourteen north-south phototrig traverse lines were planned, spaced about 40

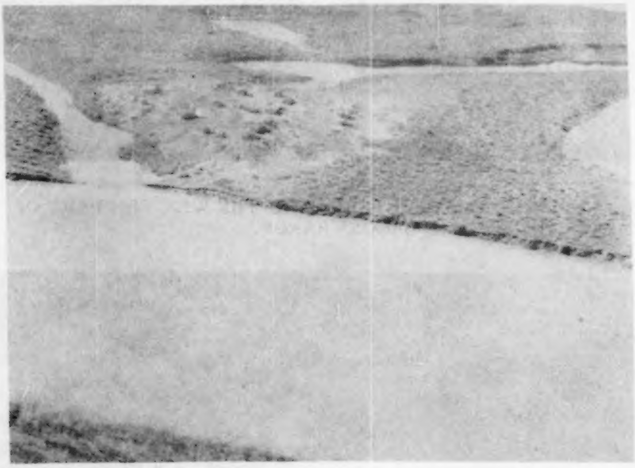

FIG. 9.—ARCTIC NATIVE VILLAGE. ONE OF THE FEW SETTLEMENTS IN THE BROOKS RANGE.

miles apart. To accomplish this control, three field parties, each consisting of four engineers and two field assistants supported by two helicopters and a fixed-wing airplane, entered the area on May 24, 1956. Less than 2 months later, the traverse lines were completed, and the parties were moved to another assignment in Alaska.

Immediately after the completion of the field work the data were shipped to the Denver office for use in compilation of the maps. The interior horizontal control was completed by stereotemplets as originally planned. Next, the fourteen photo-trig lines were computed using distances obtained from the stereotemplet solution in combination with vertical angles obtained in the field. Nine of these lines checked within 10 ft, three between 10 ft and 20 ft, one between 20 ft and 30 ft, and one, 57 ft. Vertical bridges, twelve models in length, were then set up to span the distance between phototrig lines, using the Twin-

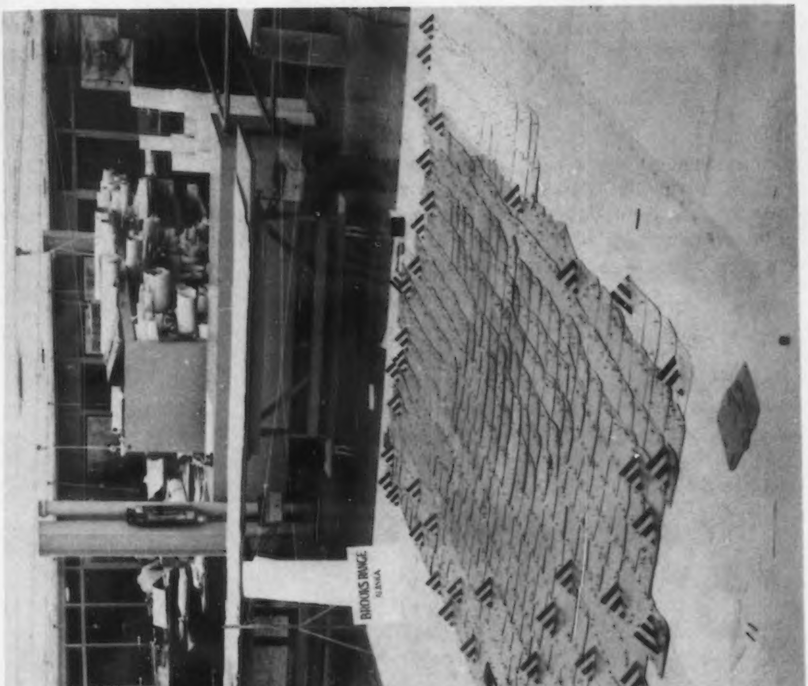

FIG. 10.—STEREOTEMPLET SETUP OF A PORTION OF THE BROOKS RANGE PROJECT

plex and ER-55 instruments. The area was then compiled with the ER-55 photogrammetric plotter at a scale of 1:63,360. Next, the compilation material was processed through the final cartographic phases and printed, resulting in 1:250,000-scale published maps.

CONCLUSIONS

The current topographic mapping program in Alaska has been a joint effort of the USGS, the USCGS and the Department of Defense. The new and enterprising techniques developed for efficient mapping of great blocks of Alaska's terrain are now available for possible application to Antarctica or any other area in the world where mapping frontiers still exist.

PROCEEDINGS PAPERS

The technical papers published in the past year are identified by number below. Technical-division sponsorship is indicated by an abbreviation at the end of each Paper Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Pipeline (PL), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways and Harbors (WW), divisions. Papers sponsored by the Department of Conditions of Practice are identified by the symbols (PP). For titles and order coupons, refer to the appropriate issue of "Civil Engineering." Beginning with Volume 82 (January 1956) papers were published in Journals of the various Technical Divisions. To locate papers in the Journals, the symbols after the paper number are followed by a numeral designating the issue of a particular Journal in which the paper appeared. For example, Paper 2270 is identified as 2270(ST9) which indicates that the paper is contained in the ninth issue of the Journal of the Structural Division during 1959.

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FEBRUARY: 1933(HY2), 1934(HY2), 1935(HY2), 1936(SM1), 1937(SM1), 1938(ST2), 1939(ST2), 1940(ST2), 1941(ST2), 1942(ST2), 1943(ST2), 1944(ST2), 1945(HY2), 1946(PO1), 1947(PO1), 1948(PO1), 1949(PO1), 1950(HY2)[c], 1951(SM1)[c], 1952(ST2)[c], 1953(PO1)[c], 1954(CO1), 1955(CO1), 1956(CO1), 1957(CO1), 1958(CO1), 1959(CO1).

MARCH: 1960(HY3), 1961(HY3), 1962(HY3), 1963(IR1), 1964(IR1), 1965(IR1), 1966(IR1), 1967(SA2), 1968(SA2), 1969(ST3), 1970(ST3), 1971(ST3), 1972(ST3), 1973(ST3), 1974(ST3), 1975(ST3), 1976(WW1), 1977(WW1), 1978(WW1), 1979(WW1), 1980(WW1), 1981(WW1), 1982(WW1), 1983(WW1), 1984(SA2), 1985(SA2)[c], 1986 (IR1)[c], 1987(WW1)[c], 1988(ST3)[c], 1989(HY3)[c].

APRIL: 1990(EM2), 1991(EM2), 1992(EM2), 1993(HW2), 1994(HY4), 1995(HY4), 1996(HY4), 1997(HY4), 1998 (SM2), 1999(SM2), 2000(SM2), 2001(SM2), 2002(ST4), 2003(ST4), 2004(ST4), 2005(ST4), 2006(PO2), 2007 (HW2)[c], 2008(EM2)[c], 2009(ST4)[c], 2010(SM2)[c], 2011(SM2)[c], 2012(HY4)[c], 2013(PO2)[c].

MAY: 2014(AT2), 2015(AT2), 2016(AT2), 2017(HY5), 2018(HY5), 2019(HY5), 2020(HY5), 2021(HY5), 2022(HY5), 2023(PL2), 2024(PL2), 2025(PL2), 2026(PP1), 2027(PP1), 2028(PP1), 2029(PP1), 2030(SA3), 2031(SA3), 2032(SA3), 2033(SA3), 2034(ST5), 2035(ST5), 2036(ST5), 2037(ST5), 2038(PL2), 2039(PL2), 2040(AT2)[c], 2041(PL2)[c], 2042(PP1)[c], 2043(ST5)[c], 2044(SA3)[c], 2045(HY5)[c], 2046(PP1), 2047(PP1).

JUNE: 2048(CP1), 2049(CP1), 2050(CP1), 2051(CP1), 2052(CP1), 2053(CP1), 2054(CP1), 2055(CP1), 2056 (HY6), 2057(HY6), 2058(HY6), 2059(IR2), 2060(IR2), 2061(PO3), 2062(SM3), 2063(SM3), 2064(SM3), 2065 (ST6), 2066(WW2), 2067(WW2), 2068(WW2), 2069(WW2), 2070(WW2), 2071(WW2), 2072(CP1)[c], 2073(IR2)[c], 2074(PO3)[c], 2075(ST6)[c], 2076(HY6)[c], 2077(SM3)[c], 2078(WW2)[c].

JULY: 2079(HY7), 2080(HY7), 2081(HY7), 2082(HY7), 2083(HY7), 2084(HY7), 2085(HY7), 2086(SA4), 2087 (SA4), 2088(SA4), 2089(SA4), 2090(SA4), 2091(EM3), 2092(EM3), 2093(EM3), 2094(EM3), 2095(EM3), 2096 (EM3), 2097(HY7)[c], 2098(SA4)[c], 2099(EM3)[c], 2100(AT3), 2101(AT3), 2102(AT3), 2103(AT3), 2104(AT3), 2105(AT3), 2106(AT3), 2107(AT3), 2108(AT3), 2109(AT3), 2110(AT3), 2111(AT3), 2112(AT3), 2113(AT3), 2114(AT3), 2115(AT3), 2116(AT3), 2117(AT3), 2118(AT3), 2119(AT3), 2120(AT3), 2121(AT3), 2122(AT3), 2123(AT3), 2124(AT3), 2125(AT3).

AUGUST: 2126(HY8), 2127(HY8), 2128(HY8), 2129(HY8), 2130(PO4), 2131(PO4), 2132(PO4), 2133(PO4), 2134 (SM4), 2135(SM4), 2136(SM4), 2137(SM4), 2138(HY8)[c], 2139(PO4)[c] 2140(SM4)[c].

SEPTEMBER: 2141(CO2), 2142(CO2), 2143(CO2), 2144(HW3), 2145(HW3), 2146(HW3), 2147(HY9), 2148(HY9), 2149(HY9), 2150(HY9), 2151(IR3), 2152(ST7)[c], 2153(IR3), 2154(IR3), 2155(IR3), 2156(IR3), 2157(IR3), 2158 (IR3), 2159(IR3), 2160(IR3), 2161(SA5), 2162(SA5), 2163(ST7), 2164(ST7), 2165(SU1), 2166(SU1), 2167(WW3), 2168(WW3), 2169(WW3), 2170(WW3), 2171(WW3), 2172(WW3), 2173(WW3), 2174(WW3), 2175(WW3), 2176 (WW3), 2177(WW3), 2178(CO2)[c], 2179(IR3)[c], 2180(HW3)[c], 2181(SA5)[c], 2182(HY9)[c], 2183(SU1)[c], 2184 (WW3)[c], 2185(PP2)[c], 2186(ST7)[c], 2187(PP2), 2188(PP2).

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NOVEMBER: 2241(HY11), 2242(HY11), 2243(HY11), 2244(HY11), 2245(HY11), 2246(SA6), 2247(SA6), 2248 (SA6), 2249(SA6), 2250(SA6), 2251(SA6), 2252(SA6), 2253(SA6), 2254(SA6), 2255(SA6), 2256(ST9), 2257(ST9), 2258(ST9), 2259(ST9), 2260(HY11), 2261(ST9)[c], 2262(ST9), 2263(HY11), 2264(ST9), 2265(HY11), 2266(SA6), 2267(SA6), 2268(SA6), 2269(HY11)[c], 2270(ST9).

DECEMBER: 2271(HY12)[c], 2272(CP2), 2273(HW4), 2274(HW4), 2275(HW4), 2276(HW4), 2277(HW4), 2278 (HW4), 2279(HW4), 2280(HW4), 2281(IR4), 2282(IR4), 2283(IR4), 2284(IR4), 2285(PO6), 2286(PO6), 2287 (PO6), 2288(PO6), 2289(PO6), 2290(PO6), 2291(PO6), 2292(SM6), 2293(SM6), 2294(SM6), 2295(SM6), 2296 (SM6), 2297(WW4), 2298(WW4), 2299(WW4), 2300(WW4), 2301(WW4), 2302(WW4), 2303(WW4), 2304(HW4), 2305(ST10), 2306(CP2), 2307(CP2), 2308(ST10), 2309(CP2), 2310(HY12), 2311(HY12), 2312(PO6), 2313(PO6), 2314(ST10), 2315(HY12), 2316(HY12), 2317(HY12), 2318(WW4), 2319(SM6), 2320(SM6), 2321(ST10), 2322 (ST10), 2323(HW4)[c], 2324(CP2)[c], 2325(SM6)[c], 2326(WW4)[c], 2327(IR4)[c], 2328(PO6)[c], 2329(ST10)[c], 2330 (CP2).

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FEBRUARY: 2355(CO1), 2356(CO1), 2357(CO1), 2358(CO1), 2359(CO1), 2360(CO1), 2361(PO1), 2362(HY2), 2363(ST2), 2364(HY2), 2365(SU1), 2366(HY2), 2367(SU1), 2368(SM1), 2369(HY2), 2370(SU1), 2371(HY2), 2372(PO1), 2373(SM1), 2374(HY2), 2375(PO1), 2376(HY2), 2377(CO1)[c], 2378(SU1), 2379(SU1), 2380(SU1), 2381(HY2)[c], 2382(ST2), 2383(SU1), 2384(ST2), 2385(SU1)[c], 2386(SU1), 2387(SU1), 2388(SU1), 2389(SM1), 2390(ST2)[c], 2391(SM1)[c], 2392(PO1)[c].

c. Discussion of several papers, grouped by divisions.

c. Discussion of several papers, grouped by divisions.

SU 1

PART 2

FEBRUARY 1960 – 4
VOLUME 86

NO. SU 1
PART 2

Your attention is invited

NEWS OF THE SURVEYING AND MAPPING DIVISION OF ASCE

AMERICAN SOCIETY OF CIVIL ENGINEERS
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JOURNAL OF THE SURVEYING AND MAPPING DIVISION
PROCEEDINGS OF THE AMERICAN SOCIETY OF CIVIL ENGINEERS

DIVISION ACTIVITIES

SURVEYING AND MAPPING DIVISION

Proceedings of the American Society of Civil Engineers

NEWS

February, 1960

Meetings: ASCE New Orleans Convention, March 7-11, 1960
ACSM-ASP Washington, D. C. Annual Meetings, March 21-25, 1960

EXECUTIVE COMMITTEE MEETS AT UNIVERSITY OF ILLINOIS SURVEYING CAMP, BLACKDUCK, MINNESOTA

The Executive Committee of the Surveying and Mapping Division met at Camp Rabideau, the University of Illinois Summer Surveying Camp near Blackduck, Minnesota on August 6-7, 1959. Professor Milton O. Schmidt, Chairman of the Committee, was host to the meeting. Other members attending were Mr. Earle J. Fennell, Prof. Arthur J. McNair, Capt. Franklin R. Gossett and the Board of Direction contact member, Dean Weston S. Evans. The pleasant surroundings and the hospitality of Prof. and Mrs. Schmidt and their assistants were thoroughly enjoyed by all.

The principal item of business was the reorganization of the technical and administrative committees of the Division. The Committees on City Surveys and Definitions of Surveying Terms were discontinued, having completed the tasks for which they had been organized. A new committee on Sessions Programs and a revised Committee on Publications were established. A new Committee on Professional Development is being established to take over the work of the Task Committee on status of Surveying and Mapping in regard to assisting in implementation of the Board's directives concerning the accepted report of the Task Committee. It was also decided to change the names of certain standing technical committees to correspond with and to cover the recommendations of the Task Committee regarding the principal categories of Surveying and Mapping. This included changing the Committee on Control Surveys to Committee on Geodetic Surveying; Committee on Highway and Bridge Surveys to Committee on Engineering Surveying; Committee on Topographic Mapping and Photogrammetry to Committee on Cartographic Surveying; Committee on Land Surveys and Titles to Committee on Land Surveying.

Prof. Schmidt announced his resignation from the Executive Committee because of his imminent departure for a year's advanced study abroad.

Note.—No. 1960-4 is part of the copyrighted Journal of the Surveying and Mapping Division, Proceedings of the American Society of Civil Engineers, Vol. 86, SU 1, February, 1960.

CHAIRMAN OF EXECUTIVE COMMITTEE TO SPEND SABBATICAL LEAVE ABROAD

Milton O. Schmidt, Professor of Civil Engineering, University of Illinois, and Chairman of the Executive Committee of the Surveying and Mapping Division left with his family the latter part of August for Switzerland where he will be at the Swiss Federal Institute of Technology in Zurich from September 1959 to September 1960. A National Science Foundation Science Faculty Fellowship makes this possible. While abroad he plans to visit a number of leading European institutes of technology which have superior programs of research and instruction in geodesy and also the facilities of several governmental mapping bureaus.

Due to this absence, Professor Schmidt resigned as chairman of the Executive Committee and as a member of the committee which normally would have expired October 1960. Earle J. Fennell has assumed the chairmanship.

NEW COMMITTEE PROPOSED BY EXECUTIVE COMMITTEE

At the meeting at Camp Rabideau, Minnesota, the Executive Committee moved to create a new permanent committee to implement the action taken by the Board of Direction at the Los Angeles Convention in February relative to the place of surveying and mapping in the civil engineering profession. To date, the work of implementing that action has been done by the members of the Task Committee on Status of Surveying and Mapping; Brother B. Austin Barry, Geo. D. Whitmore and Alfred O. Quinn. These men are to be invited to serve as the nucleus of the new committee. The motion contemplates a continuing study of the problems in such fields as ethics, education in the field of surveying and mapping, registration and membership qualifications. Committee and subcommittee membership could include people from all areas of the Society's membership. The Chairman of the Executive Committee would like to hear from members who are especially interested in these critical matters.

INTERNATIONAL GROUP CONSIDERS MAPPING OF ANTARCTICA

The Working Group on Cartography of the Special Committee on Antarctic Research, International Committee of Scientific Unions held a meeting at Canberra, Australia, March 2-6, 1959. The United States member who attended the meeting was George D. Whitmore, (F) ASCE. Mr. Whitmore is Chief Topographic Engineer of the U. S. Geological Survey. The Working Group on Cartography developed a series of recommendations pertaining to the coordination of mapping activities of the several countries. At this meeting, Mr. Whitmore submitted the United States' plan for the topographic mapping of Antarctica.

SURVEYS IN ANTARCTICA

Another ASCE member, William H. Chapman, was assigned to Byrd Station, Antarctica, for a period of 18 months. He will accompany scientific traverse parties and will conduct geodetic surveys to determine positions of major peaks and other landmarks for mapping control.

Two other engineers are being assigned to Antarctica in the near future for shorter periods. Their task will be to determine astronomic positions of identifiable ground features, a project similar to that being accomplished by Mr. Chapman. One engineer, Louis J. Roberts, will operate from McMurdo Naval Air Facility over the Victoria Land Plateau for a trip of about 1000 miles. The other engineer, Warren T. Borgeson, will be based aboard an icebreaker operating along the coast of the Amundsen and Bellingshausen Seas. Roberts is also a member of ASCE.

BRITISH COMMONWEALTH SURVEY OFFICERS' CONFERENCE

The British Commonwealth Survey Officers' Conference was held at Cambridge, England, from August 17-26. Among several representatives from the United States were ASCE members, Rear Admiral H. Arnold Karo, Director of the Coast and Geodetic Survey, George D. Whitmore, Chief Topographic Engineer of the Geological Survey, and David L. Mills, Chief, Geodetic Division, Army Map Services.

In addition to the exchange of information about surveying matters, the conference gave Mr. Whitmore an opportunity of discussing with our colleages abroad, the activities of the Task Committee on the Status of Surveying and Mapping, and the resultant ASCE action of officially recognizing that several professional categories within the field of surveying and mapping are engineering in nature.

Some of the non-United States participants in the conference arranged their itinerary so that they would go through the United States en route to their homes, and thereby were able to visit surveying and mapping agencies in Washington, D. C.

MEETING OF THE ASCE COMMITTEE ON HIGHWAY AND BRIDGE SURVEYS HELD AT CHICAGO ON JULY 11, 1959

The meeting was called to order by the chairman at 9:00 a.m. in the second-floor conference room of Caffarellas, directly across Cicero Avenue from the Main Terminal Building at Midway Airport at Chicago, Illinois. Present were: O. R. Bosso, P. A. Hakman, R. H. Dodds, C. A. Rothrock, and M. O. Schmidt, chairman.

Intensive review of Mr. Bosso's last draft of Construction Surveys (for Bridges) which will be Chapter 7 of the proposed ASCE Manual on Highway and Bridge Surveys was the principal item of business. It was agreed that the final draft of that portion of Chapter 7 dealing with triangulation should be made ready for presentation to the Executive Committee of the Surveying and Mapping Division at its special summer meeting at the University of Illinois Summer Surveying Camp on August 6-7 in accordance with policy of the Division.

The committee recommended that the Executive Committee of the Division name R. H. Dodds Acting Chairman of the Committee on Highway and Bridge Surveys during the forthcoming absence from the country on sabbatical leave of Chairman Schmidt from September 1959 to September 1960.

PIPELINE LOCATION COMMITTEE MEETING

A meeting of the ASCE Pipeline Division Task Committee on Pipeline Location was held in Tulsa, Oklahoma on July 9-10, 1959. In attendance were J. F. Schaffer, E. H. Schmidt, R. H. Dodds, M. O. Schmidt, and E. O. Scott, chairman. This committee is a cooperative group representing both the Pipeline and the Surveying and Mapping Divisions. Messrs. R. H. Dodds and M. O. Schmidt represent the latter division.

The immediate task of the Committee is to prepare a Manual on Pipeline Location. Most of the business transacted at this meeting was related to clarifying the responsibilities for writing various portions of the manual and to encouraging the participation of various pipeline location specialists in the forthcoming regional meetings of ASCE.

DEVELOPMENTS IN PHOTOGRAMMETRIC AND SURVEYING INSTRUMENTS

Submitted by ASCE Committee on Research and Development

Recently developed wide-angle cameras with high resolution and low distortion characteristics include the Fairchild T-12 Planigon, the Wild Aviogon and the Zeiss Pleogon.

Of still more recent origin are the new super-wide-angle aerial cameras which are now available. Whereas the wide-angle cameras were designed for an angle coverage of about 90°, the new super-wide-angle camera has an angle of coverage of about 120°.

Investigation is now proceeding toward design of compatible plotting instrumentation for the super-wide-angle cameras.

A new type of radar instrument has been developed for reconnaissance mapping. A continuous picture of the terrain is photographed on two radar screens carried in a fast-moving aircraft. One screen registers electronically the terrain data to the left of the aircraft; the other registers the terrain data to the right. The terrain data is wiped on to a continuously moving film, the speed of which is synchronized with the speed of the aircraft. The resulting radar photographs not only are at larger scale than conventional radar pictures widely used in World War II, but also present far greater detail than was possible with the older "Plan Position Indicator, or PPI" pictures that were widely used for reconnaissance purposes. The new side-looking radar is not dependent on daylight as the images are not optical, and can be used at night, over clouds, or under the most severe meteorological conditions. For this reason, side-looking which presents mapping data at the rate of speed of the aircraft itself, looms as an attractive means for mapping Antarctica and other polar regions.

Tellurometers are being widely used for control operations for topographic mapping. These instruments are distance-measuring devices which interpret the phase relationship of a modulated radio signal response in such a manner that distances between the master and remote units may be determined. Although the Tellurometer makes possible trilateration procedures, it is also used in the execution of traverse. Closures of second-order accuracy have been attained consistently and costs of Tellurometer traverse have averaged about 50 per cent of costs for comparable transit and tape traverse. Accurate measurements can be made of distances from about 1/2 mile to about 30 miles.

AMERICAN CONGRESS ON SURVEYING AND MAPPING ESTABLISHES A PROFESSIONAL STATUS COMMITTEE

In June, President George C. Bestor of the American Congress on Surveying and Mapping established a Professional Status Committee. The primary task before the committee is a study, from the viewpoint of ACSM and particularly the surveyor, of the report of the ASCE Task Committee on Status of Surveying and Mapping. The first action of the new ACSM Committee has been to undertake a review of the "Classification Chart for Surveying and Mapping," contained in the ASCE Task Committee Report.

The members of the ACSM Professional Status Committee are as follows:

Lester C. Higbee, W. & L. E. Gurley Co., Troy, N. Y., Chairman

Howard J. Teas, F. ASCE, Teas and Steinbrenner, Malverne, N. Y., Secretary

James L. Bell, James L. Bell & Associates, Kansas City, Kansas

Ralph M. Berry, F. ASCE, Dept. of Civil Engineering, University of Michigan, Ann Arbor, Michigan

John I. Davidson, Topographic Division, U. S. Geological Survey, Washington, D. C.

Winfield H. Eldridge, M. ASCE, Department of Civil Engineering, University of Illinois, Urbana, Illinois

Granville K. Emminizer, U. S. Coast and Geodetic Survey, Washington, D. C.

Victor H. Ghent, Cross and Ghent, Alexandria, Virginia

A. J. Hoskinson, U. S. Coast and Geodetic Survey, Washington, D. C.

William A. White, M. ASCE, Exec. Secretary, State Council of Civil Engineers and Land Surveyors, Sacramento, California

Francis L. Witkege, F. ASCE, Topographic Division, U. S. Geological Survey, Washington, D. C.

The newsletter will contain further information on activities of this committee in subsequent issues.

TASK COMMITTEE ON STATUS OF SURVEYING AND MAPPING PRESENTS REPORT ON IMPLEMENTATION

A report of particular interest to all persons engaged in the surveying and mapping fields of civil engineering was presented to the Executive Committee at its Blackduck meeting. The recommendations contained in this report and other especially significant portions are as follows:

"As requested by the Executive Committee of the S. & M. Division, the Task Committee has continued its studies and discussions concerning the professional status of surveying and mapping.

"Serious conversations have been held with various individuals and private organizations engaged in surveying and mapping in an attempt to further analyze and to establish implementing procedures to put into operation the Board's February 1959 approval of the professional aspects of surveying and mapping. ASCE Headquarters in New York has written to various government agencies (Corps of Engineers, Department of Interior), State Departments (Massachusetts, New York, and New Jersey), and to various individuals, advising them of the Board's action and requesting their cooperation. The report of the Task

Committee and notice of the action of the ASCE Board of Direction have been published in Civil Engineering and other magazines, and editorial comment in Engineering News-Record has been particularly helpful in advising fellow engineers and others on the professional status of surveying and mapping.

"Your Task Committee is appreciative of the work which has been done since the Board's action in February, but we recognize that continued positive action must be taken if our objectives are to be achieved in the not too distant future. We make the following recommendations with the hope that the Executive Committee will endorse and approve the fundamental concepts and recommend appropriate action by ASCE.

"1. ASCE must undertake and continue a program of basic education. Engineers and the public generally must be acquainted with the current status of surveying and mapping and with present developments and capabilities of modern surveying and mapping methods and techniques. We believe that ASCE membership at large is generally poorly informed on the surveying and mapping story, and they must be told in many ways about the present 'state of the art.'

"We recommend that the Surveying and Mapping Division's publication committee actively solicit from outstanding engineers engaged in this work, articles concerning the work which they are performing and the potentialities and uses of surveying and mapping techniques that may be used to solve a wide variety of problems. These papers should be published in Civil Engineering and ASCE Proceedings, and be made available for other publications.

"2. Liaison should be established between ASCE headquarters and the Surveying and Mapping Division to assure a uniform and well organized program for the continuance of the letters from ASCE to government officials, State Highway Departments, ASCE members, and others to advise them of the ASCE stand on the professional status of surveying and mapping. These people must be kept informed by ASCE as to their responsibilities and the responsibilities of others in implementing this program. (See 'Suggested Basic Rules' attached.) The current Task Committee would be willing to perform the liaison work if that be the desire of the Executive Committee.

"3. Since an important part of the implementation program lies in the hands of the Registration Boards of the various States, we feel that the ASCE Committee on Registration should be asked to study (with the S. & M. Division) certain important surveying-mapping problems that now arise. This work may well involve correspondence and/or contact with the various Registration Boards to assist them in their review of the ASCE decision on Surveying and Mapping.

"4. ASCE must recognize that the Professional Engineering concept represents a distinct change in the way in which many of the private surveying and mapping (particularly photogrammetric) organizations are now operated. Therefore, there is a need for a period of transition in order to clear the status of surveying and mapping with the various state Registration Boards, and to provide time for each company engaged in this work to satisfy internal problems as required by the application of professional engineering license laws and methods of operation in the various states. In addition, considerable effort must be made in the education of potential clients, fellow engineers, and non-engineers engaged in this work with regard to the professional engineering status of surveying and mapping. To act as a guide in making the required change to professional engineering, we suggest that the attached 'Suggested Basic Rules' be approved and forwarded by ASCE to the various companies engaged in this work and the potential users of these services.

"5. In order to provide an orderly method for estimating costs and fees for surveying and mapping work, we recommend that further studies be undertaken to establish an acceptable means and method for presenting such information to engineers and the general public. To date, two methods have been suggested:

"a. To establish median estimated costs of selected representative surveying and mapping projects, such as highway mapping. This would require the acceptance of general specifications such as those established by the 'Reference Guide Outline—Specifications for Aerial Surveys and Mapping by Photogrammetric Methods for Highways—1958' published by the Department of Commerce, Bureau of Public Roads, and the imposition of certain conditions such as length and width of the project, scale, contour interval, and the condition of the terrain—flat, rolling, or rough.

"b. To establish median rates for personnel (professional, subprofessional, and technicians) to be assigned to a given project.

"At the present time it is not clear just which of these procedures should be adopted as a substitute for competitive-price bidding. The Task Committee is still at work on the matter and most certainly welcomes the views of all concerned."

These recommendations and basic rules have the unqualified endorsement of the Executive Committee of the Surveying and Mapping Division. The Executive and Assistant Secretaries of ASCE have also studied the report and are most favorably disposed toward its conclusions.

"Attachment:

"Suggested Basic Rules for Professional Responsibility in the Performance of Surveying-Mapping Services.

"These concepts are to be considered supplementary to the fundamental accepted procedural rules for negotiating for professional engineering services of whatever nature. They are intended to form a basis for the elimination of competitive-price bidding formerly used for many surveying-mapping professional engineering services.

"The title indicated it is our presently 'suggested' rules-of-the-road program, but when fully developed and made final, the term 'suggested' will, of course, be deleted.

"A. In order to be a Professional' there are certain standards which must be attained and maintained. Four principal hallmarks of a professional are:

"1. A professional requires certain high standards of competence.
"2. A professional requires certain high standards of integrity.
"3. A professional is unbiased.
"4. A professional stands ready at all times to render public service.

"B. In order to conform with these standards, organizations and persons engaged in surveying and mapping work must be prepared to accept the following responsibilities in connection with their activities in surveying and mapping work:

"1. Abide by the Engineer's Code of Ethics—a copy of the ASCE code is attached.

"2. All work must be accomplished in conformance with plans and specifications which are mutually agreed upon by the engineer and his client, or if plans and specifications are not practical, then the work must be of a quality that will render it fully satisfactory for its intended uses.

"3. Each surveying-mapping organization should be prepared to provide competent consulting personnel who can discuss surveying and mapping problems with potential clients.

"4. A registered professional engineer must be in responsible charge of the surveying and mapping work.

"5. Maps and survey notes should bear the engineer's seal to indicate compliance with the plans and specifications prepared for each project, or that the work is fully satisfactory quality-wise for the intended uses. The affixing of the engineer's seal is recognized as imparting considerable responsibility and liability, and each engineer is reminded that his own professional reputation is at stake each time he places his engineer's seal on a survey or a map. This means that the engineer is responsible for the content and accuracy of the maps and surveys produced under his direction.

"C. In the initial phases of the contemplated changes, effective and firm self-policing will be required to insure that all practitioners properly interpret and abide by the engineer's code of ethics."